Asian Research Journal of Mathematics

7(1): 1-15, 2017; Article no.ARJOM.36503
ISSN: 2456-477X

Effect of Vertical Magnetic Field on the Onset of Double Diffusive Convection in a Horizontal Porous Layer with Concentration Based Internal Heat Source

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Authors' contributions

This work was carried out in collaboration between all authors. Authors CIC and LE designed the study and performed the analysis. Author EA presented the plots/tables and interpreted the results. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/36503
Editor(s):
(1) Jitender Singh, Guru Nanak Dev University, Punjab, India.
Reviewers:
(1) Nor Fadzillah Mohd Mokhtar, Universiti Putra Malaysia, Malaysia.
(2) S. R. Mishra, SOA University, India.
(3) Hulin Huang, Nanjing University of Aeronautics and Astronautics, China.
Complete Peer review History: http://www.sciencedomain.org/review-history/21378

Original Research Article

Received: 30th August 2017
Accepted: 6th October 2017
Published: 13th October 2017

Abstract

This study considers the effects of concentration based internal heat and vertical magnetic field on the onset of double diffusive convection in a horizontal porous layer using normal mode analysis. The normal mode analysis is used to find solutions for the fluid variables, the critical wave number and the critical Rayleigh number for the onset of convection with free-free boundaries. The results obtained are displayed graphically and in tables. The results show that the concentration based internal heat, γ, hastens the onset of instability while the magnetic field, Ha, and solutal Rayleigh number, Rs, delays the onset of instability in the system for stationary and oscillatory convections. The influence of Lewis number, Le, and porosity, ε, is also presented.

Keywords: Double diffusive convection; concentration based internal heat; magnetic field; normal mode analysis.

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1 Introduction

The study of convective motions in the presence of two buoyancy driven components is an important phenomenon in the field of convection. The well-known Rayleigh-Benard type problem has been the subject of several research interest from which convective instability limit of a fluid heated from below has been determined. In the study of the onset of double diffusive convective in a porous medium due to temperature and concentrative gradients, the relationship between the fluxes and the driving potentials are of intricate nature and has got applications in the behavior of fluids in earth's crust, oil reservoir modeling, biomechanics, nuclear waste repository, metallurgy crystal production, migration of moisture through air contained in fibrous insulation. Excellent reviews in the study of double diffusive convection in porous media can be found in [1,2,3,4,5,6,7].

Linear stability analysis was used to study the onset criterion of marginal and oscillatory convection, [8]. The onset of double diffusive reaction convection in a fluid layer heated and salted from below subject to chemical equilibrium on the boundaries with extension to nonlinear stability analysis was considered in [9]. The modified Darcy-Maxwell model for the momentum equation was used to study the onset of double diffusive convection [10]: the study noted that there is a competition between the process of viscoelasticity and diffusions that causes the convection to set in through oscillatory rather than stationary.

There are situation in which convection can be triggered by internal heat source, for example convective flux in the earth's crust is basically due to the internal heating of the multicomponent fluids saturating it. Nonlinear stability of double diffusive convection with throughflow and concentration based internal heat source was considered in [11]. The study observed that for downgrade throughflow, when the Peclet number is high, the effect of concentration based internal heat is significant. The effect of internal heat source on double diffusive convection in a couple stress fluid saturated horizontal anisotropic porous layer using linear and nonlinear stability analyses was studied in [12]. The results show that heat transport increased and mass transport decreased with increase in internal heat source parameter. Double diffusive Hadley-Prats flow with concentration based internal heat source using linear and nonlinear stability analyses was investigated in [13]. They showed that an increase in both the heat source and mass flow results in destabilization. Reviews on internal heat source in porous media can also be found in early studies, [14,15,16,17,18]. There is interest also, in the study of magnetohydrodynamics (MHD) flow and internal heat transfer on the performance of many systems using electrically conducting fluids; for example in geothermal energy extractions and nuclear reactors. Heat transfer effects on MHD flow through a porous medium was considered in [19]. The effect of internal heat generation or absorption on MHD free convection flow of an incompressible, electrically conducting fluid was found to be significant, [20]; while [21,22] considers MHD flow of a visco-elastic fluid in a porous medium and showed that heat source and magnetic field have considerable effect on the flow regime.

Although some work on double diffusive convection in porous medium with concentration based internal heat source is available, to the best of our knowledge, attention has not been given to the study of combined effect of concentration based internal heat source and vertical magnetic field on the onset of double diffusive convection in a horizontal porous layer. Therefore, the main aim of this study is to investigate the effect of vertical magnetic field on the onset of double diffusive convection in a horizontal porous layer with concentration based internal heat source using normal mode analysis. This constitutes an important addition to the study of double diffusion in porous media.

2 Mathematical Formulation

We consider an infinite horizontal fluid-saturated porous layer with concentration based internal heat source confined between two parallel horizontal planes located at $z^* = \frac{h}{2}$ and $z^* = -\frac{h}{2}$, respectively, see Fig. 1. A Cartesian frame of reference (x^*, y^*, z^*) is chosen with the x^*-axis along the horizontal plate, and the z^*-axis vertically upwards, while the gravitational force $\vec{g}$ acts vertically downwards. Adverse temperature and concentration gradient are applied across the porous layer in such a way that the lower plate is maintained at temperature $T_1(= T_0 + \delta T)$ and concentration $C_1(= C_o + \delta C)$; while the upper plate is held at temperature T_2 and concentration C_2 respectively. Also, $T_0 = (T_1 + T_2)/2$, $C_o = (C_1 + C_2)/2$ with $T_1 > T_2$, $C_1 > C_2$.

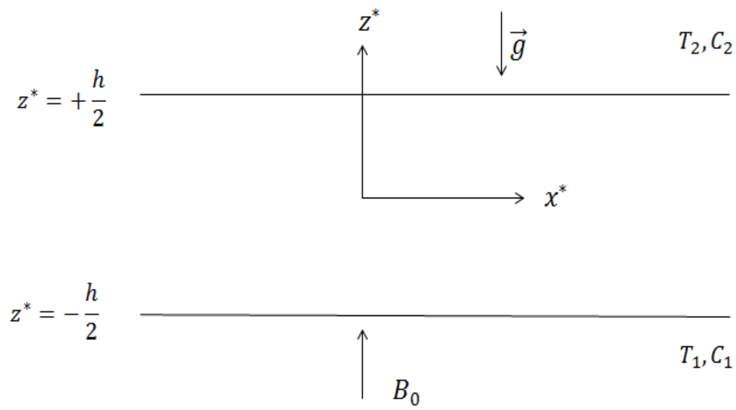

Fig. 1. Geometry of the flow region

A magnetic field of strength $\vec{B}_0$ is applied vertically upwards in the z^* direction in which the induced magnetic field is neglected on the account that the magnetic Reynolds number is small. Further, the internal heat source is assumed linearly with respect to solute concentration. Assuming the Oberbeck-approximation due to the density variations and employing the Darcy model and adopting the Lorentz force, the governing equations are [2,5].

$$\vec{\nabla}^*.\vec{\mathrm{V}}^* = 0 \tag{1}$$

$$\vec{\nabla}P^* + \frac{\mu}{K}\vec{\mathrm{V}}^* + \rho(T^*, C^*)\vec{g}\,\vec{k} - \sigma_c(\vec{\mathrm{V}}^* \times \vec{\mathrm{B}}^*) \times \vec{\mathrm{B}}^* = 0 \tag{2}$$

$$A\frac{\partial T^*}{\partial t^*} + (\vec{\mathrm{V}}^*.\vec{\nabla}^*)T^* = \propto_T \Delta^* T^* + Q(C^* - C_0) \tag{3}$$

$$\emptyset\frac{\partial c^*}{\partial t^*} + (\vec{\mathrm{V}}^*.\vec{\nabla}^*)C^* = D_c\Delta^* C^* \tag{4}$$

$$\rho(T^*, C^*) = \rho_0[1 - \beta_T(T^* - T_0) + \beta_c(C^* - C_0)] \tag{5}$$

with boundary conditions:

$$w^* = 0,\ T^* = T_0 + \frac{(T_1+T_2)}{2},\ \ c^* = c_0 + \frac{(c_1+c_2)}{2}\ \text{at}\ z^* = -\frac{1}{2} \tag{6}$$

$$w^* = 0,\ T^* = T_0 - \frac{(T_1+T_2)}{2},\ \ c^* = c_0 - \frac{(c_1+c_2)}{2}\ \text{at}\ z^* = +\frac{1}{2} \tag{7}$$

In the above equations $A = (\rho c_P)_m/(\rho c_P)_f$, $\propto_T = K_T/(\rho c_P)_f$, $Q = {}^{\beta}\!/_{(\rho c_P)_f}$, $\vec{\mathrm{B}}^* = (0,0,B_0)$ is the magnetic field and where $\vec{\mathrm{V}}^* = (u^*, v^*, w^*)$ is the velocity, μ is the dynamic viscosity, κ is the permeability, K_T is the thermal diffusivity, $\vec{k}$ is the unit vector in the z^* direction, T^* is the temperature, C^* is the solute concentration, β_T is the thermal expansion coefficient, β_c is the solute expansion coefficient, D_c is solute concentration diffusivity P^* is the pressure, g is the gravitational acceleration, β is the a constant of proportionality, arising from the internal heat source, c_p is the heat capacity, ρ_0 is the reference density $T_0{}^*$ is the reference temperature, C_0 is the reference solute concentration and ϕ porosity parameter. Also, $\vec{B}^*$ is the magnetic field strength, σ_c is the electric conductivity. The subscripts m , f denote medium and fluid respectively.

Using the following non-dimensional variables;

$$(x,y,z) = \frac{1}{h}(x^*,y^*,z^*),\ t = \frac{\propto_T}{Ah^2}\,t^*,\ \vec{V} = \frac{h}{\alpha_T}\vec{V}^*,\ P = \frac{k}{\mu\alpha_T}(P^* + \rho_0 g z^*),\ \overline{\vec{\nabla}} = h\overrightarrow{\nabla^*},$$
$$T = \sqrt{Ra}\left(\frac{T^*-T_0}{\delta_T}\right), C = \frac{C^*-C^0}{\delta_c}\ ,\ \varepsilon = \emptyset/A\ ,\ \delta_T = T_2 - T_{1,}\ \delta_c = c_{1-}c_2\ , \tag{8}$$

the governing equations (1-5) and the boundary conditions (equations (6) and (7)) in dimensionless form become,

$$\vec{\overline{\nabla}}.\vec{V} = 0 \tag{9}$$

$$\vec{\nabla}P + \vec{V} + Ha^2(u,v,o) - \sqrt{R_a}\,T\,\vec{k} + RsC\vec{k} = 0 \tag{10}$$

$$\frac{\partial T}{\partial t} + \sqrt{Ra}\,(\vec{V}.\nabla)T = \Delta T + \sqrt{Ra}\,\gamma c \tag{11}$$

$$\varepsilon\frac{\partial c}{\partial t} + (\vec{V}.\nabla)T = \frac{1}{Le}\Delta c \tag{12}$$

$$w = 0,\ T = +\frac{1}{2},\ C = +\frac{1}{2}\ \text{ at }\ z = -\frac{1}{2} \tag{13}$$

$$w = 0,\ T = -\frac{1}{2},\ C = -\frac{1}{2}\ \text{ at }\ z = +\frac{1}{2} \tag{14}$$

The dimensionless quantities are

$$Ra = \sqrt{\frac{\rho_0 g\beta_T Kh\delta_T}{\mu\kappa_T}} = \text{thermal Rayleigh number}\ ,\ Rs = \frac{hK\rho_0 g\beta_c(\delta c)}{\mu\alpha_T} = \text{solutal Rayleigh number}$$

$$Ha = \sqrt{\frac{K\sigma_e\,Bo^2}{\mu}} = \text{Hartmann number (magnetic parameter)},\ Le = \frac{\propto_T}{D_c} = \text{Lewis number}$$

$$\gamma = \frac{h^2 Q(\delta_c)}{\propto_T(\delta_T)} = \text{Internal heat source parameter},\ \varepsilon = {}^{\phi}/_{A} = \text{normalized porosity parameter.}$$

2.1 Basic state

To study the stability of the system, we assume that the basic state denoted by $\big(P_b(z), T_b(z), C_b(z)\big)$ is quiescent and superimpose small perturbations on the basic state in the form

$$\vec{V} = (0,0,0) + \vec{v},\ P = P_b(z) + p,\ T = T_b(z) + \theta, C =\ C_b(z) + \varphi \tag{15}$$

Where p, θ, φ denote the quantities at the perturbations and the subscript, b, refers to the basic state. Substituting equation (15) into equations (9) - (12) and the boundary conditions (13) and (14) yield the basic state equations

$$\frac{dP_b}{dz} = \sqrt{Ra}T_b + Rs\ C_b \tag{16a}$$

$$\frac{d^2T_b}{dz^2} + \gamma\sqrt{Ra}C_b = 0 \tag{16b}$$

$$\frac{d^2C_b}{dz^2} = 0 \tag{16c}$$

Subject to

$$T_b = C_b = +\frac{1}{2} \quad at \quad z = -\frac{1}{2} \tag{17a}$$

$$T_b = C_b = -\frac{1}{2} \quad at \quad z = +\frac{1}{2} \tag{17b}$$

Solving Equations (16a-c) subject to conditions (17a-b) yields

$$T_b = \frac{1}{24}(-24\,z - \gamma z + 4\gamma z^3) \tag{18a}$$

$$C_b = -z \tag{18b}$$

$$P_b = \int\left(-\sqrt{Ra}T_b + RsC_b\right)dz \tag{18c}$$

2.2 Linearized equations

Substituting equations (16) and (18) into the governing equations, we obtain the linearized equations as

$$\vec{\vec{\nabla}}.\vec{\mathrm{V}} = 0 \tag{19}$$

$$\vec{\mathrm{V}} \quad = -\nabla \mathrm{p} \quad - \mathrm{Ha}^2(u, v, o) + \sqrt{Ra}\theta\vec{k} - Rs\varphi\vec{k} \tag{20}$$

$$\frac{\partial\theta}{\partial t} + \sqrt{Ra}f(z)w = \Delta\theta + \gamma\sqrt{\mathrm{Ra}}\varphi \tag{21}$$

$$Le\varepsilon\frac{\partial\varphi}{\partial t} = \Delta\varphi + w \tag{22}$$

$$w = 0 = \theta = \varphi \quad \text{ on } \quad z = \pm\frac{1}{2} \tag{23}$$

where we have neglected the nonlinear terms in the system and $f(z) = -\frac{\partial T_b}{\partial z} = \frac{1}{24}(24 + \gamma - 12\gamma z^2)$ is the temperature gradient.

Now, the pressure term in Equation (20) is eliminated by taking double curl and keeping only the z -component. This equation yield

$$\nabla^2\mathrm{w} + \mathrm{Ha}^2\frac{\partial^2\mathrm{w}}{\partial \mathrm{z}^2} = \sqrt{\mathrm{Ra}}\nabla_{\mathrm{h}}{}^2\theta - \mathrm{Rs}\nabla_{\mathrm{h}}{}^2\varphi \tag{24}$$

$$w = 0 = \frac{\partial^2\mathrm{w}}{\partial \mathrm{z}^2} \quad \text{ on } \quad z = \pm\,\frac{1}{2}$$

where $\nabla_{\mathrm{h}}{}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian in the horizontal plane

3 Onset of Convection

Equations (24), (21), (22) expressed as in the order

$$\nabla^2 w + Ha^2 \frac{\partial^2 w}{\partial z^2} = \sqrt{Ra}\nabla_h{}^2\theta - Rs\,\nabla_h^2\varphi \tag{25a}$$

$$\frac{\partial\theta}{\partial t} + \sqrt{Ra}\;\; f(z)w = \nabla^2\theta + \gamma\sqrt{Ra}\varphi \tag{25b}$$

$$Le\varepsilon\frac{\partial\varphi}{\partial t} - Lew = \nabla^2\varphi \tag{25c}$$

now subject to the boundary conditions,

$$w = \frac{\partial^2 w}{\partial z} = \theta = \varphi = 0 \quad on\;\; z = \pm\frac{1}{2} \tag{26}$$

constitute a linear boundary value problem that is solved using the method of normal mode analysis.

Now, we assume a time dependent periodic disturbance of the form [23]

$$\begin{pmatrix} w(x,y,z,t) \\ \theta(x,y,z,t) \\ \varphi(x,y,z,t) \end{pmatrix} = e^{\Omega t}\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} f(x,y) \tag{27}$$

where, $\Omega = \sigma + iw$ is the growth rate and is in general complex (with σ, w real) and $f(x,y,)$ is a horizontal plane tilting the plane $(x,y,)$ periodically. Substituting equation (27) into the eigenvalue problem (25) yields

$$\begin{pmatrix} D^2 - a^2 - {H_a}^2 D^2 & a^2\sqrt{Ra} & -a^2 Rs \\ f(z)\sqrt{Ra} & D^2 - a^2 - \Omega & \gamma\sqrt{Ra} \\ Le & O & D^2 - a^2 - Le\mathcal{E}\Omega \end{pmatrix}\begin{pmatrix} W \\ \Theta \\ \Phi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{28}$$

$$W = \frac{d^2 w}{dz^2} = \Theta = \Phi = 0,\;\; z = \pm\frac{1}{2}$$

$\nabla_h{}^2 f + a^2 f = 0$, a, is the wave number, [24], and $D = \frac{d}{dz}$ $D^2 = \frac{d^2}{dz^2}$

Now, assuming the lowest eigen solutions of $W(z)$, $\Theta(z)$, $\Phi(z)$ of the form,

$$[W(z), \Theta(z), \Phi(z)] = [W_0,\; \Theta_0, \Phi_0\,] \sin\pi z \tag{29}$$

where $w_0, \theta_0, \Phi_{0,}$ are constants. The eigenvalue problem (28) becomes

$$\bar{\mathrm{A}}\,\bar{X} = \bar{0} \tag{30}$$

where

$$\bar{\mathrm{A}} = \begin{pmatrix} J + Ha^2\pi^2 & -\,a^2\sqrt{Ra} & a^2 Rs \\ -f(z)\sqrt{Ra} & \mathrm{J} + \Omega & -\,\gamma\sqrt{\mathrm{Ra}} \\ -Le & 0 & J + \varepsilon\, Le\Omega \end{pmatrix}$$

$\bar{X} = (w_0, \Theta_0, \Phi_0)^T$, $\bar{O} = (O,O,O)^T$ and $\mathrm{J} = a^2 + \pi^2$

The solvability of the eigenvalue problem given in equation (30) requires that $|A| = 0$. That is, for non-trivial solution of system (30), we obtain the expression for the thermal Rayleigh number as;

$$Ra = \frac{(J+\Omega)(J^2+\pi^2 Ha^2\, J+ a^2 Le\, Rs+ J(J+ \pi^2 Ha^2)Le\mathcal{E}\,\Omega}{a^2(Le\gamma+F(J+ Le\mathcal{E}\,\Omega)} \tag{31}$$

In general, the growth rate $\Omega\ = \sigma_r + i\omega$ is a complex quantity. The system is stable of $\sigma_r\ < 0$ and unstable if $\sigma_r\ > 0$.

3.1 Marginal stationary state

For the validity of principle of exchange of instabilities to hold and for marginal stationary convection to occur we set $\Omega = 0$. Setting $\Omega = 0$ in equation (31) yields the Rayleigh number for the stationary convection from which the conditions for stationary convection can be determined as

$$Ra(st) = \frac{J(J^2+\pi^2\, Ha^2\, J+a^2 Le\, Rs)}{a^2(Le\,\gamma+\frac{1}{24}(24+\gamma)J\,)} \tag{32}$$

Now, the minimum wave number (critical wave number) for the onset of stationary convection is obtained by putting $a = a_c$ and then minimizing equation (32) using

$$\frac{\partial Ra(st)}{\partial a_c^2} = 0 \tag{33}$$

Equation (33) after simplification yields the following 8th order polynomial in a_c

$$a_4\, a_c^8 + a_3\, a_c^6 + a_2\, a_c^4 + a_1\, a_c^2 + a_0 = 0 \tag{34}$$

where

$$a_0 = -24\,(1 + Ha^2)\,(24Le\,\gamma + \pi^2\,(24 + \gamma))\pi^6$$

$$a_1 = -48(1 + Ha^2)\pi^6$$

$$a_2 = 24\pi^2(24Le(3\pi^2 + Le\,Rs)\gamma - Ha^2((24 + \gamma)\pi^4 - 24Le\,\gamma\,\pi^2))$$

$$a_3 = 48\,(24Le\,\gamma + (24 + \gamma)\pi^2\,)$$

$$a_4 = 24(24 + \gamma)$$

The solution of equation (34) yields eight roots of which only one root is real and is equal to π. Substituting $a_c = \pi$ in equation (32) yields the critical Rayleigh number for stationary convection in the presence of magnetic and internal heat source parameters as

$$Ra^c(st) = \frac{24\pi^2(2\pi^2(2+Ha^2)+Le\,Rs)}{12\,Le\,\gamma+\pi^2(24+\gamma)} \tag{35}$$

In the absence of internal heat source, γ, and magnetic field parameter, Ha, equation, (35) reduces to

$$Ra^c(st) = 4\pi^2 + Le\,Rs \tag{36}$$

which is the same result obtained by [25]. Further for $Rs = 0$, equation (36) becomes

$$Ra^c(st) = 4\pi^2 \tag{37}$$

which corresponds to the classical result of [26] and [27] for single component convection in a porous layer.

3.2 Marginal oscillatory convection

For marginal oscillatory convection, the real part of Ω, must be equal to zero (i.e. $\sigma_r = 0$). Therefore, setting $\Omega = i\omega$ in equation (31) we obtain the expression for the marginal oscillatory Rayleigh number as

$$Ra(OS) = \frac{(J+i\omega)\,(\pi^2 Ha^2+J)J+a^2 Le\,Rs+i\omega\,Le\,\varepsilon(\pi^2 Ha^2+J)}{a^2(Le\,\gamma+FJ+i\omega F\,Le\,\varepsilon)}$$

$$= \Delta_1 + i\omega\,\Delta_2 \qquad (38)$$

Where:

$$\Delta_1 = \frac{1}{a^2}\left(\frac{a_{33}(a_{11}J-a_{22}\omega^2)+\omega^2(a_{11}+a_{22}J)a_{44}}{a_{33}^2+a_{44}^2\,\omega^2}\right) \qquad (39)$$

$$\Delta_2 = \frac{1}{a^2}\left(\frac{a_{33}(a_{11}+a_{22}J)-a_{44}(a_{11}J-a_{22}\omega^2)}{a_{33}^2+a_{44}^2\omega^2}\right) \qquad (40)$$

$$J = a^2 + \pi^2$$

$$a_{11} = \pi^2 Ha^2 J + J^2 + a^2\,Le\,Rs = (\pi^2 Ha^2 + J)J + a^2\,Le\,Rs$$

$$a_{22} = Le\varepsilon(\pi^2 Ha^2 + J)$$

$$a_{33} = Le\,\gamma + FJ = Le\,\gamma + \tfrac{1}{24}(24+\gamma)$$

$$a_{44} = FLe\varepsilon = \tfrac{1}{24}(24+\gamma)Le\varepsilon$$

Since $Ra(os)$ is a physical quality it must be real. Therefore for marginal oscillatory stability $\omega \neq 0$, hence $\Delta_2 = 0$. By setting $\Delta_2 = 0$ in equation (38), and after simplification gives an expression for the frequency of oscillation as

$$\omega^2 = \frac{a_{11}a_{44}J-a_{33}(a_{11}+a_{22}J)}{a_{22}a_{44}} \qquad (41)$$

Now, substituting $\Delta_1 = 0$ in equation (38), we get,

$$Ra(os) = \frac{(1+Le\varepsilon)(a^2+\pi^2)(a^2+\pi^2+\pi^2Ha^2)}{a^2F\,Le\varepsilon} + \frac{Rs}{F\varepsilon} \qquad (42)$$

where $F = \frac{24+\gamma}{24}$

In the absence of Ha and γ,

$$Ra(os) = \frac{(1+Le\varepsilon)}{Le\varepsilon}\frac{(a^2+\pi^2)^2}{a^2} + \frac{Rs}{\varepsilon} \qquad (43)$$

Equation (43) corresponds to the result of [25] and [28]. Equation (42) gives the oscillatory neutral Rayleigh number; $Ra(os)$ with critical oscillatory Rayleigh number, $Ra^c(os)$ as

$$Ra^c(os) = \frac{(1+Le\varepsilon)(2\pi^2)(2\pi^2+\pi^2+Ha^2)}{\pi^2\,F\,Le\varepsilon} + \frac{Rs}{F\varepsilon} \qquad (44)$$

4 Discussion of Results

The effects of concentration based internal heat source and magnetic field on the onset of double diffusive convection in a horizontal porous layer heated from below is analyzed using normal mode analysis. The expression for the Rayleigh numbers for the stationary and oscillatory convections are given in Equations (32) and (42), respectively. The effects of magnetic field parameter, Ha and the internal heat source parameter, γ, on the stationary thermal Rayleigh number, $Ra(st)$ are shown in Tables 1 and 2. It is observed from Table 1 that in the presence of solute and concentration based internal heat the Rayleigh number for stationary convection decreases. More so, increase in the heat parameter further decreases the Rayleigh number. The heat parameter therefore has a destabilizing effect on the system. In the absence of magnetic field parameter, Ha, this result is in agreement with the result reported in [16].

Table 1. Thermal Rayleigh number values for $Ha = 2$, $Le = 1$, and variations in internal heat and Rs

$Rs = 0, \gamma$=0,		$Rs = 10, \gamma = 1$		$Rs = 10, \gamma$=5	
a	$Ra(st)$	a	$Ra(st)$	a	$Ra(st)$
2.5	143.395	2.5	138.982	2.5	101.016
2.6	138.025	2.6	134.349	2.6	98.0949
2.7	133.317	2.7	130.295	2.7	95.563
2.8	129.18	2.8	126.743	2.8	93.368
2.9	125.54	2.9	123.626	2.9	91.466
3.0	122.334	3.0	120.89	3.0	89.8205
3.1	119.508	3.1	118.489	3.1	88.4009
3.2	117.02	3.2	116.384	3.2	87.1811
3.3	114.831	3.3	114.541	3.3	86.139
3.4	112.909	3.4	112.934	3.4	85.2557

Table 2 shows that increase in magnetic parameter, Ha, (from 0 to 2) with the heat parameter fixed, leads to increased values of Ra for stationary convection. This is an indication that the onset of instability in the system is delayed. Therefore, the presence of solute and increased magnetic field parameter stabilizes the system.

Table 2. Thermal Rayleigh number values for $\gamma = 1$, $Le = 1$, and variations in Ha and Rs

$Rs = 0, Ha = 0$		$Rs = 0, Ha = 1$		$Rs = 10, Ha = 2$	
a	$Ra(st)$	a	$Ra(st)$	a	$Ra(st)$
2.5	41.5746	2.5	60.7315	2.5	153.395
2.6	40.9088	2.6	59.1649	2.6	148.025
2.7	40.3911	2.7	57.8418	2.7	143.317
2.8	40.0038	2.8	56.7308	2.8	139.18
2.9	39.7317	2.9	55.8056	2.9	135.54
3.0	39.5624	3.0	55.0445	3.0	132.334
3.1	39.4854	3.1	54.4291	3.1	129.508
3.2	39.4917	3.2	53.9438	3.2	127.02
3.3	39.574	3.3	53.5753	3.3	124.831
3.4	39.7255	3.4	53.3124	3.4	122.909

Fig. 2 shows the influence of magnetic field parameter, Ha on the thermal Rayleigh number, Ra for fixed values of $Rs = 10, \gamma = 5$ and $Le = 1$. It is observed that increase in the magnetic field increases the thermal Rayleigh number, for the stationary mode. This implies that magnetic field stabilizes the system.

Fig. 3 depicts the effect of magnetic field parameter, Ha, on oscillatory convection for fixed values of $\gamma = 0.5$, $Le = 10$ and $\varepsilon = 0.5$, where it is evident that increases in magnetic field increases the thermal

Rayleigh number for oscillatory convection. Hence the magnetic field parameter, Ha, delays the onset of instability in the system.

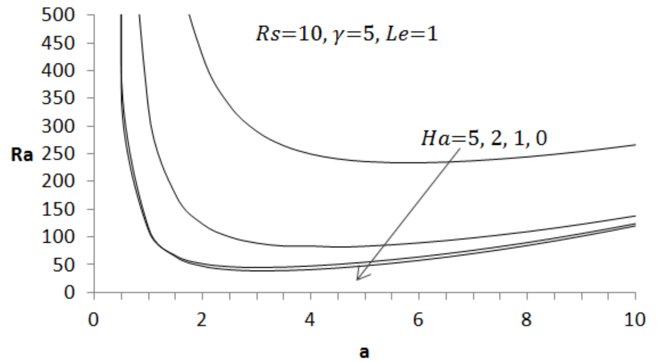

Fig. 2. Variation of thermal Rayleigh number for various values of the magnetic parameter, Ha for stationary convection

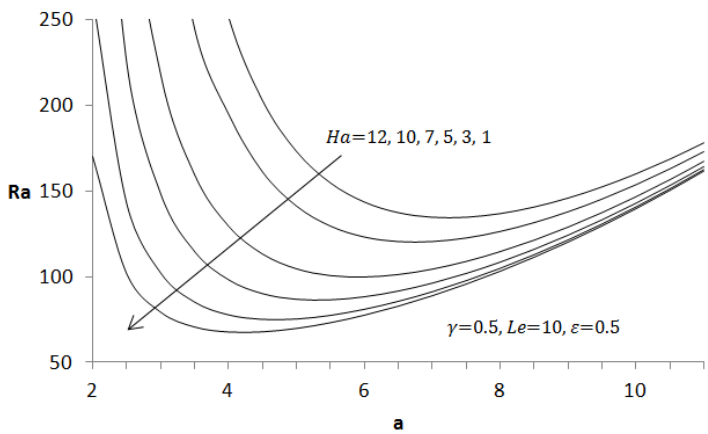

Fig. 3. Variation of thermal Rayleigh number for various values of the magnetic parameter, Ha, for oscillatory convection

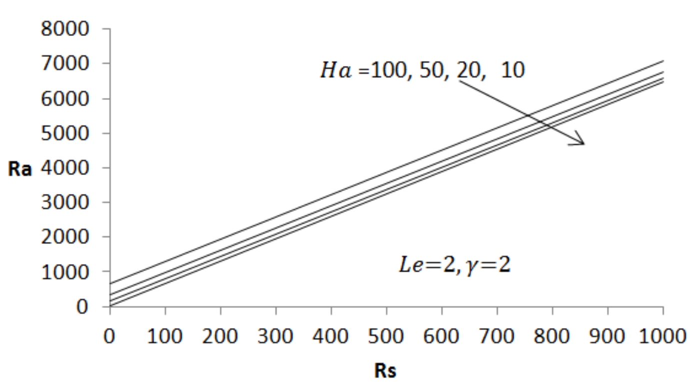

Fig. 4. Variation of the thermal Rayleigh number with solutal Rayleigh number for various values of magnetic field in stationary convection

Fig. 4 shows a linear relationship between the thermal Rayleigh number, Ra, and the solutal Rayleigh number, Rs, for variations in the magnetic field parameter, Ha. Increase in the magnetic field parameter increases the thermal Rayleigh number, which indicates that the magnetic field parameter stabilizes the system for stationary convection. This is because the magnetic field couples together with temperature and concentration field to inhibit double diffusive effects. Also, Fig. 5 presents a linear relationship between the thermal Rayleigh number and the solutal Rayleigh number for variations in internal heat source, γ, in stationary convection. The variations show that the thermal Rayleigh number decreases with increase in internal heat source parameter, indicating that the internal heat source has the tendency to destabilize the system. In the absence of magnetic field, this is in agreement with earlier results of [16]. The instability is more pronounced at higher values of the solutal Rayleigh number.

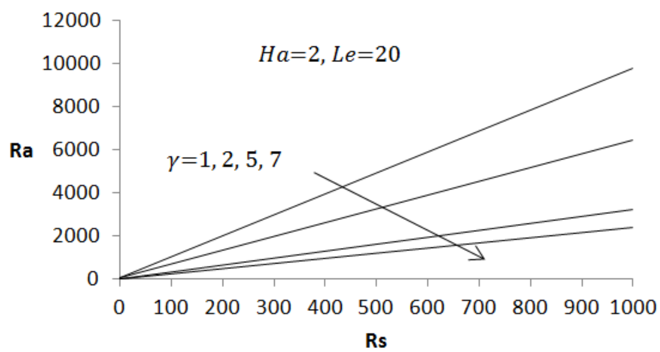

Fig. 5. Variation of the thermal Rayleigh number with solutal Rayleigh number for various values of internal heat source parameter in stationary convection

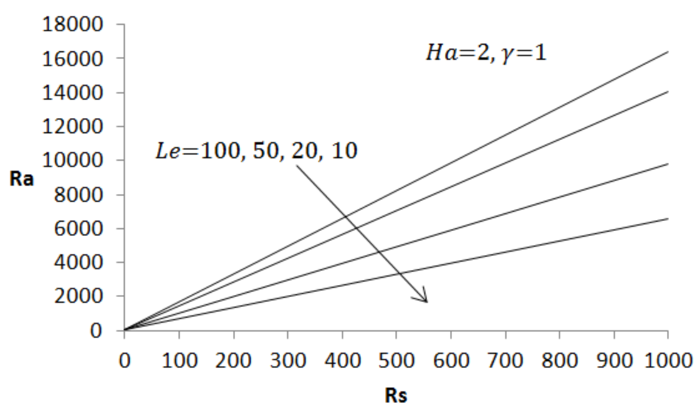

Fig. 6. Variation of the thermal Rayleigh number with solutal Rayleigh number for various values of the Lewis number in stationary convection

Fig. 6 shows a linear relationship between the thermal Rayleigh number and the solutal Rayleigh number for variations in Lewis number, Le. Increase in the Lewis number increases the thermal Rayleigh number which in essence has the effect of stabilizing the system for stationary mode. In this case, increase in the solutal Rayleigh number promotes the stability of the system. Fig. 7 depicts the effect of the Lewis number, Le, on the system for oscillatory convection. It is observed that increase in the Lewis number decreases the critical thermal Rayleigh number, indicating that the Lewis number destabilizes the system in oscillatory mode. This is so because the diffusivity of heat is greater than that of the solute for $Le > 1$. Moreso, the results shows the fact that the oscillatory instability is caused by the difference in the rates of heat and mass.

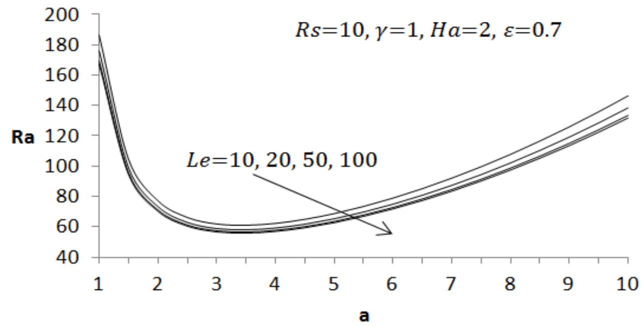

Fig. 7. Variation of the thermal Rayleigh number for various values of the Lewis number in oscillatory convection

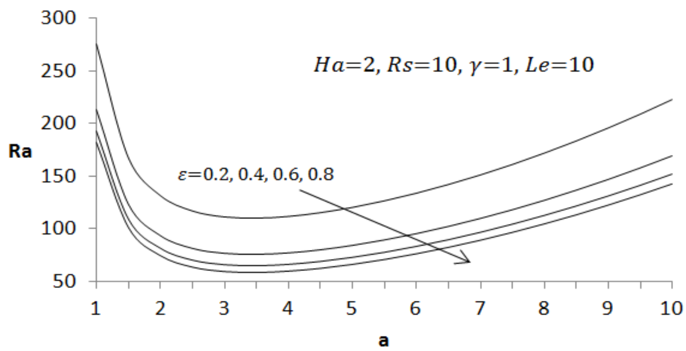

Fig. 8. Variation of the thermal Rayleigh number for various values of the porosity parameter in oscillatory convection

The effect of porosity, ε, on the oscillatory convection is shown in Fig. 8. Increase in the porosity parameter decreases the thermal Rayleigh number. This shows that increasing porosity has the effect of destabilizing the system. This effect is more pronounced for small values of the porosity parameter. Fig. 9 depicts the effect of internal heat parameter on the system for oscillatory convection. We find that the thermal number decreases as the internal heat parameter increases, indicating that increase in internal heat parameter hastens the onset of instability for oscillatory convection.

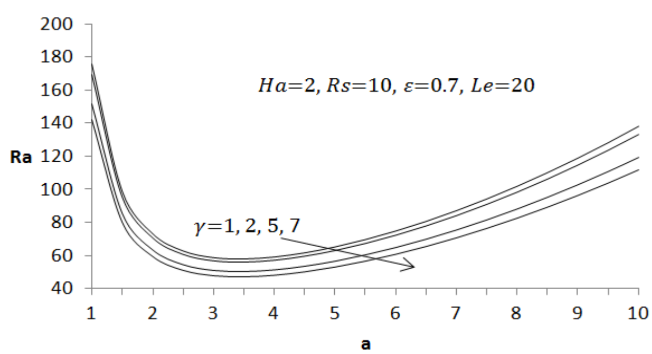

Fig. 9. Variation of the thermal Rayleigh number for various values of the internal heat source in oscillatory convection

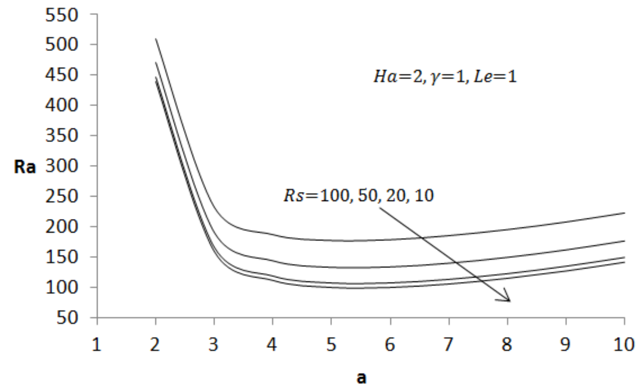

Fig. 10. Variation of the thermal Rayleigh number for various values of the solutal Rayleigh number in stationary convection

Figs. 10 and 11 depicts the effect of solutal Rayleigh number, Rs, on stationary and oscillatory convection, respectively. It is observed in Fig. 10 that for fixed values of the parameters, Ha, γ and Le, increase in the solutal Rayleigh number leads to increase in the thermal Rayleigh number, Ra. This indicates that the solutal Rayleigh number stabilizes the system. Fig. 11, shows that for fixed values of Ha, γ, ε and Le, (for oscillatory convection), the solutal Rayleigh number exerts stability on the system. In the absence of Ha and γ, this finding is in agreement with [25].

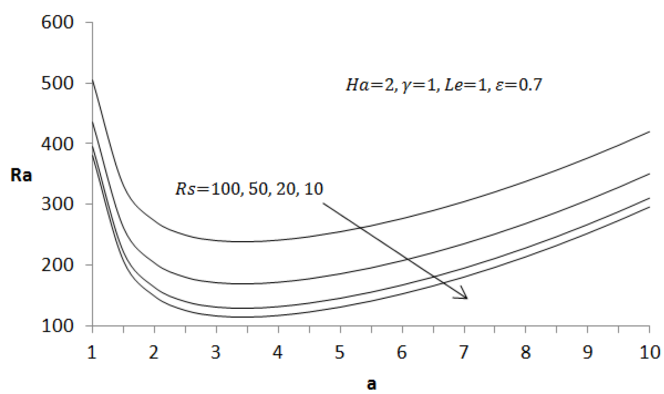

Fig. 11. Variation of the thermal Rayleigh number for various values of the solutal Rayleigh number in oscillatory convection

5 Conclusion

In this article, we have presented the results of the combined effects of concentration based internal heat source, vertical magnetic field, Lewis number and porosity on double diffusive convection in a horizontal porous layer using normal mode analysis. The result of our study shows that increase in the internal heat parameter, γ, hastens the onset of instability for stationary and oscillatory mode. Increase in the porosity, ε, destabilizes the system for oscillatory mode, while increase in the magnetic field and solutal Rayleigh number increases the Rayleigh number for both stationary and oscillatory modes which in effect leads to the stability of the system. The presence of solute and increase in the magnetic field further stabilizes the system. Increase in the Lewis number ,Le, stabilizes the system for stationary mode while it destabilizes the system for oscillatory mode.

Competing Interests

Authors have declared that no competing interests exist.

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