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Preservation of Stability for Reduced Order Model of Large Scale Sys[tems Using Di](www.sciencedomain.org)fferentiation Method

D. K. Sambariya¹*[∗]* **and Hem Manohar**¹

¹*Department of Electrical Engineering, Rajasthan Technical University, Kota 324010, India.*

Authors' contributions

This work was carried out in collaboration between both authors. Author DKS designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors DKS and HM managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, higher order single-input and single-output models are considered for reduction using differentiation method. The analysis of higher order models is complex, and time consuming. The application of model order reduction is necessitated to reduce the system with its core properties. The analysis of a low order system is easy, and fast process. The higher order systems are reduced using differentiation method and the results are compared with original and the reduced systems in literature. The performance comparison of the proposed reduced order model and original as well as systems in literature have been considered in terms of settling time, rise time, peak and peak time. The application of proposed differentiation method is applied to discrete system and found equally good to retain the stability in reduced order model.

^{}Corresponding author: E-mail: dsambariya 2003@yahoo.com;*

Keywords: Reduced Order Model (ROM) techniques; differentiation method; stability equation eethod; stability array method.

1 Introduction

The necessity of the model order reduction (MOR)is to make easy steps to analysis systems for dynamic response. The prevalent systems are of a large order; therefore the analysis of these systems is difficult and time consuming for control system engineers. The reduced model order (ROM) system is always stable as against an original stable system [1]. The differentiation equation method is comparable in performance with the mixed mathematical methods using pade approximation method, genetic algorithm and Particle swarm optimization techniques, stability array method, stability equation method [2]. The differentiation method tends to give better response as compared to above methods [3].

The stability array method is based on the Routh stability criterion. In this method, the reduced order transfer function is [d](#page-14-0)irectly obtained from the Routh stability array and no algorithm is required [4]. In the [st](#page-14-1)ability equation method, the pole-zero pattern of a large scale original systems is the source of construction ROM models [5, 6].

The particle swarm optimization (PSO) and genetic algorithm (GA) based MOR techniques are used for the order reduction. The PSO and GA are search techniques and effectively proposed in [7]. The diffe[re](#page-14-2)ntiation equation method is based on the differentiation of the transfer function. This method does not make use of any stability [c](#page-14-3)r[it](#page-14-4)erion but always lead to the stable reduced model order for stable system [8]. It provides appreciable improvement in the values of settling time, rise time, peak- time and peak value of reduced order systems as compared to other mathemati[ca](#page-14-5)l methods [1].

In this paper, the higher order system are reduced to reduced order models and examined for system stability. It is found tha[t t](#page-14-6)he stability of original system is preserved in its reduced order model. The paper is organized in four sections. The way of problem formulation is described in section 2. The differ[en](#page-13-0)tiation method used for model order reduction is mentioned in this section. The use of proposed differentiation method is described by considering three different continuous systems and one on discrete systems in section 3 . Finally, the paper is concluded in section 4 and followed by references.

2 Problem Formul[a](#page-2-0)tion

Consider a single-input single-output higher order system of order n and is represented by Eqn. 2.1.

$$
G(s) = \frac{N(s)}{D(s)} = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}
$$
\n(2.1)

where, a_i and b_i are constants for $i = 1, 2, ..., n$. If r represents a reduced o[rder](#page-1-0) as of lesser order than *n*, then, the reduced order model of the system in Eqn. 2.1 is represented as in Eqn. 2.2. The important and principal requirement of the reduced order model is to posses all important specifications of the original system.

$$
G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \dots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + \dots + d_r}
$$
(2.2)

where, c_r and d_r are unknown constants and are to be determined by using differentiation method.

2.1 Differentiation method

The proposed method is based on differentiation of polynomials. The reciprocals of the numerator and denominator of the higher order transfer functions are differentiated successively many times to yield the coefficients of the reduced order transfer function. The reduced polynomials are reciprocated back and normalized. The straight forward differentiation is discarded because it has a drawback that zero with large modulus tend to be approximated than those with a small modulas [8, 9].

2.1.1 Algorithm of differentiation method

- *•* T[he](#page-14-6) [re](#page-14-7)ciprocal of the transfer function(s) is taken.
- *•* The reciprocated transfer function is differentiated to get desired order.
- *•* The reduced transfer function is again reciprocated.
- *•* Finally, the steady state correction is applied to the reduced order system.

3 Numerical Examples

The proposed method is explained with illustrated numerical examples compared better than other method. Considering some single-input single-output high order transfer function are reduced by differentiation method.

3.1 Example-1

Considering a single-input single-output transfer function of 8*th* order system as shown in Eqn. 3.1 [2].

$$
G_8(s) = \frac{35s^7 + 1086s^6 + 13285s^5 + 82402s^4 + 278376s^3 + 511812s^2 + 482964s + 194480}{s^8 + 33s^7 + 437s^6 + 3017s^5 + 11870s^4 + 27470s^3 + 37492s^2 + 28880s + 9600} \tag{3.1}
$$

[3.](#page-14-0)1.1 ROM by methods in literature

The 2*nd* order reduced order of Eqn. 3.1 is solved using Routh array method and is reproduced as in Eqn. 3.2 [2].

$$
G_2(s) = \frac{334828.5s + 194480}{20123.7s^2 + 18116.2s + 9600}
$$
(3.2)

The 2*nd* order reduced order of Eqn. [3.1](#page-2-1) solved using mixed method by [10] is given in Eqn. 3.3.

$$
G_2(s) = \frac{944795700.576s + 392072337.6168}{26994240s^2 + 145555200s + 193536000}
$$
(3.3)

The 2*nd* order reduced order of Eqn. [3.](#page-2-1)1 solved using stability equatio[n m](#page-14-8)ethod and is giv[en](#page-2-2) in Eqn. 3.4 [5].

$$
G_2(s) = \frac{482964s + 194480}{34194s^2 + 28880s + 9600}
$$
\n(3.4)

The 2*nd* order reduced order of Eqn. 3.1 [so](#page-2-1)lved using Routh approximation method and is given in Eqn. [3.5](#page-2-3) [by](#page-14-3) [6].

$$
G_2(s) = \frac{17.03s + 6.8646}{s^2 + 1.02s + 0.3366}
$$
\n(3.5)

3.1.2 ROM using proposed method

The reciprocal of Eqn. 3.1 is taken as following Eqn. 3.6.

$$
\hat{G}_8(s) = \frac{35 + 1086s + 13285s^2 + 82402s^3 + 278376s^4 + 511812s^5 + 482964s^6 + 194480s^7}{1 + 33s + 437s^2 + 3017s^3 + 11870s^4 + 27470s^5 + 37492s^6 + 28880s^7 + 9600s^8} \tag{3.6}
$$

T[he](#page-2-1) differentiation of the Eqn. 3.6 is take upto get[ting](#page-3-0) the 2^{nd} order reduced model of it. The obtained 2^{nd} order reduced model is presented in Eqn. 3.7.

$$
\hat{G}_2(s) = \frac{347734080 + 980179200s}{26994240 + 145555200s + 193536000s^2} \tag{3.7}
$$

It is the 2*nd* order reduced model by differentiation met[hod](#page-3-1) of the reciprocal of the original system. Therefore, the Eqn. 3.7 is reciprocated to get 2*nd* order reduced model of the system in Eqn. 3.1 and is shown by Eqn. 3.8.

$$
G_2(s) = \frac{347734080s + 980179200}{26994240s^2 + 145555200s + 193536000}
$$
(3.8)

The reduced model pr[odu](#page-3-2)ced by differentiation method suffers with steady state error; therefore, the steady state error correction is compensated by multiplying the correction factor (*k*) defined as following in Eqn. 3.9.

$$
k = \frac{Steadystatefactor of original system}{Steadystate factor of reduced ordersystem}
$$
\n(3.9)

In this case the [SS f](#page-3-3)or original system is $SS_O = 194480/9600 = 20.25$ and the SS for reduced order system is $SS_{G2} = 980179200/193536000 = 5.06$. The correction factor $k = SS_{O}/SS_{G2}$ $20.25/5.06 = 4$. The 2^{nd} order reduced model after correction is given as following in Eqn. 3.10

$$
G_2(S) = \frac{1390936320s + 3920716800}{26994240s^2 + 145555200s + 193536000}
$$
(3.10)

3.1.3 Simulation results

To verify the applicability of the reduced model obtained by proposed method and models by methods in literature, the step response performance is compared. It is clear that the step response of original system in Eqn. 3.1, ROM in Eqn. 3.2 - Eqn. 3.5 and proposed ROM in Eqn. 3.10 is shown in Fig. 1. Moreover, the information of the step response is included in Table 1.

Table 1. Step response information of original system, ROM systems in literature an[d pr](#page-2-1)oposed ROM [of](#page-2-4) system [in](#page-2-5) Example-1

System	Rise Time(s)	Settling Time (s)	Peak	Peak Time (s)
Original $(Eqn. 3.1)$	1.0727	1.5826	20.3884	2.2557
Prop. $(Eqn. 3.10)$	0.8752	1.5689	20.2476	3.0622
RA (Eqn. 3.2)	1.3883	7.0542	24.2156	3.4963
Mixed (Eqn. 3.3)	1.0864	1.8702	20.2491	3.5028
SE (Eqn. 3.4)	1.9609	9.4676	22.4589	4.7204
RA (Eqn. 3.5)	1.7975	8.5154	22.0293	4.3201

Fig. 1. Step response of original system, ROM as in Eqns. 3.2 - 3.5 and proposed ROM using differentiation equation method as in Eqn. 3.10

3.2 Example-2

Considering a single-input single-output transfer function of 4*th* order system a[s show](#page-3-4)n in Eqn. 3.11 [11, 12].

$$
G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}
$$
\n(3.11)

[3.2](#page-14-9)[.1](#page-14-10) ROM by methods in literature

This transfer function is solved by mixed mathematical method using pade approximation technique and is given in Eqn. 3.12 [13].

$$
G_2(s) = \frac{0.891s + 1.0569}{s^2 + 2.0356s + 1.0569}
$$
\n(3.12)

The 2^{nd} reduced mo[del u](#page-4-0)s[ing](#page-14-11) stability equation method and GA, is given in Eqn. 3.13 [11].

$$
G_2(s) = \frac{0.7442575s + 0.6991576}{s^2 + 1.45771s + 0.6997}
$$
\n(3.13)

The 2*nd* order reduced model using particle swarm optimization technique and is gi[ven i](#page-4-1)[n Eq](#page-14-9)n. 3.14 [14].

$$
G_2(s) = \frac{2.9319s + 7.8849}{3.8849s^2 + 11.4839s + 07.8849}
$$
(3.14)

The 2*nd* order reduced model using genetic algorithm technique and is given in Eqn. 3.15 [14][.](#page-4-2)

$$
G_2(s) = \frac{5.2054s + 8.989}{6.6076s^2 + 14.8941s + 8.989} \tag{3.15}
$$

The 2*nd* order reduced model using Genetic Algorithm as given in Eqn. 3.16 [15].

$$
G_2(s) = \frac{0.7645s + 1.689}{s^2 + 2.591s + 1.689}
$$
\n(3.16)

The 2nd order reduced model using conventional method and is given in [Eqn](#page-5-0). [3.](#page-14-12)17 [16].

$$
G_2(s) = \frac{s + 24.0096}{s^2 + 27.0096s + 24.0096}
$$
\n(3.17)

The 2*nd* order reduced model using Big-Bang Big-Crunch method and s given [in E](#page-5-1)[qn.](#page-15-0) 3.18 [17].

$$
G_2(s) = \frac{0.9315s + 1.6092}{s^2 + 2.75612s + 1.6092}
$$
\n(3.18)

The 2*nd* order reduced model using Big-Bang Big-Crunch method and is given in Eqn. [3.19](#page-5-2) [\[18](#page-15-1)].

$$
G_2(s) = \frac{0.6965s + 0.6858}{s^2 + 1.428s + 0.6858}
$$
\n(3.19)

This transfer function is also solved by using stability equation method and continu[ed fr](#page-5-3)[acti](#page-15-2)on method and is given in Eqn. 3.20 [19].

$$
G_2(s) = \frac{0.6997(s+1)}{s^2 + 1.45771s + 0.6997}
$$
\n(3.20)

This transfer function solved [by u](#page-5-4)[sin](#page-15-3)g particle swarm optimized Eigen spectrum analysis is given in Eqn. 3.21 [7].

$$
G_2(s) = \frac{0.6349s + 4}{s^2 + 5s + 4}
$$
\n(3.21)

This transfer function solved by using error minimization technique is given in Eqn. 3.22 [12].

$$
G_2(s) = \frac{0.8000003s + 2}{s^2 + 3s + 2}
$$
\n(3.22)

It is function solved by using Factor division algorithm is given in Eqn. 3.23 [20].

$$
G_2(s) = \frac{0.833s + 2}{s^2 + 3s + 2}
$$
\n(3.23)

The 2*nd* order reduced model using a two-step iterative method is given [in E](#page-5-5)[qn.](#page-15-4) 3.24 [21].

$$
G_2(s) = \frac{0.7273s + 2.6497}{s^2 + 3.5975s + 2.6497}
$$
\n(3.24)

The 2nd order reduced model using optimal routh approximants through inte[gral s](#page-5-6)[qua](#page-15-5)red error minimization: computer-aided approach is given in Eqn. 3.25 [22].

$$
G_2(s) = \frac{0.764444s + 1.688910}{s^2 + 2.590980s + 1.688910}
$$
\n(3.25)

The 2*nd* order reduced model using factor division and [Eigen](#page-5-7) [sp](#page-15-6)ectrum analysis is given in Eqn. 3.26 [23].

$$
G_2(s) = \frac{0.6667s + 4}{s^2 + 5s + 4}
$$
\n(3.26)

The 2^{nd} order reduced model using improved pole clustering is given in Eqn. 3.27 [24].

$$
G_2(s) = \frac{0.8s + 2.0004}{s^2 + 3.0004s + 2.0004}
$$
\n(3.27)

The 2*nd* order reduced model using Pade approximation and clustering technique is given in Eqn. 3.28 [25].

$$
G_2(s) = \frac{-0.189762s + 4.5713}{s^2 + 4.76187s + 4.5713}
$$
\n(3.28)

The 2*nd* order reduced model using clustering, integral square error (ISE) minimization techniques [and](#page-6-0) [dom](#page-15-7)inant pole technique is given in Eqn. 3.29 [26].

$$
G_2(s) = \frac{0.74791s + 2.7692}{s^2 + 3.7692s + 2.7692}
$$
\n(3.29)

The 2*nd* order reduced model using minimiza[tion](#page-6-1) [of i](#page-15-8)ntegral square error performance indices is given in Eqn. 3.30 and 3.31 [27].

$$
G_2(s) = \frac{0.81796s + 0.78411}{s^2 + 1.64068s + 0.78411}
$$
\n(3.30)

$$
G_2(s) = \frac{0.76434s + 1.69073}{s^2 + 2.59288s + 1.69073}
$$
\n(3.31)

3.2.2 ROM using proposed method

The reciprocal of the 4*th* order system in Eqn. 3.11 is presented in the following Eqn. 3.32.

$$
\hat{G}_4(s) = \frac{1 + 7s + 24s^2 + 24s^3}{1 + 10s + 35s^2 + 50s^3 + 24s^4}
$$
\n(3.32)

The 2nd order reduced model of the reciprocat[ed sy](#page-4-3)stem (Eqn. [3.](#page-6-2)32) is given as in Eqn. 3.33.

$$
\hat{G}_2(s) = \frac{48 + 144s}{70 + 300s + 288s^2} \tag{3.33}
$$

The steady-state correction factor *k* is calculated and is $k = \frac{24/24}{144/288} = \frac{1}{0.5} = 2$ $k = \frac{24/24}{144/288} = \frac{1}{0.5} = 2$ $k = \frac{24/24}{144/288} = \frac{1}{0.5} = 2$. Taking [recip](#page-6-3)rocal of Eqn. 3.33 and multiplying by *k*; the resultant reduced order model is presented by Eqn. 3.34.

$$
G_2(s) = \frac{96s + 288}{70s^2 + 300s + 288}
$$
\n(3.34)

3.2.3 Simulation results

To verify the applicability of the reduced model obtained by proposed method and models by methods in literature, the step response performance is compared. It is clear that the step response of original system in Eqn. 3.11, ROM in Eqn. 3.12 - Eqn. 3.31 and proposed ROM in Eqn. 3.34 is shown in Fig. 2 - Fig. 6. Moreover, the information of the step response is included in Table 2.

3.3 Example-3

Considering a single-input single-output transfer function of 4*th* order system as shown in Eqn. 3.35 [28].

$$
G_4(s) = \frac{4.269s^3 + 5.10s^2 + 3.9672s + 0.9567}{4.39992s^4 + 9.0635s^3 + 8.021s^2 + 5.362s + 1}
$$
(3.35)

Fig. 2. Step response of original system (Eqn. 3.11), ROM as in Eqns. 3.12 - 3.15 and proposed ROM using differentiation equation method as in Eqn. 3.34.

Fig. 3. Step response of original system (Eqn. 3.11), ROM as in Eqns. 3.16 - 3.19 and proposed ROM using differentiation equation method as in Eqn. 3.34

Fig. 4. Step response of original system (Eqn. 3.11), ROM as in Eqns. 3.20 - 3.23 and proposed ROM using differentiation equation method as in Eqn. 3.34

Fig. 5. Step response of original system (Eqn. 3.11), ROM as in Eqns. 3.24 - 3.27 and proposed ROM using differentiation equation method as in Eqn. 3.34

3.3.1 ROM by methods in literature

The 2^{nd} order reduced model of the original system (Eqn. [3.3](#page-4-3)5) is given as in Eqn. [3.36](#page-5-6) [\[29](#page-6-4)[\].](#page-5-8)

$$
G_2(s) = \frac{0.2211s + 0.0702}{s^2 + 1.9531s + 0.1415}
$$
\n(3.36)

Fig. 6. Step response of original system (Eqn. 3.11), ROM as in Eqns. 3.28 - 3.31 and proposed ROM using differentiation equation method as in Eqn. 3.34

The 2*nd* order reduced model using Pade approximation [is gi](#page-4-3)ven in Eqn. 3.37 [30].

$$
G_2(s) = \frac{1.869s + 0.5585}{s^2 + 2.663s + 0.5838}
$$
\n(3.37)

3.3.2 ROM using proposed method

The reciprocal of the system in Eqn. 3.35 is given as following in Eqn. 3.38.

$$
\hat{G}_4(s) = \frac{4.269 + 5.10s + 3.9672s^2 + 0.9567s^3}{4.39992 + 9.0635s + 8.021s^2 + 5.362s^3 + s^4}
$$
\n(3.38)

By taking differentiation of Eqn. 3.38 [to g](#page-6-5)et 2^{nd} order reduced model a[s fol](#page-9-0)lowing in Eqn. 3.39.

$$
\hat{G}_2(s) = \frac{7.9344 + 5.7402s}{16.042 + 32.172s + 12s^2}
$$
\n(3.39)

The steady-state correction factor *[k](#page-9-0)* is calculated and is $k = \frac{0.9567/1}{5.7402/12} = \frac{0.9567}{0.4784} = 1.9998$. [Taki](#page-9-1)ng reciprocal of Eqn. 3.39 and multiplying by *k*; the resultant reduced order model is presented by Eqn. 3.40.

$$
G_2(s) = \frac{15.8672s + 11.4793}{16.042s^2 + 32.172s + 12}
$$
\n(3.40)

3.3.[3 S](#page-9-2)imula[tion](#page-9-1) results

To verify the applicability of the reduced model obtained by proposed method and models by methods in literature, the step response performance is compared. It is clear that the step response of original system in Eqn. 3.35, ROM in Eqn. 3.36 - Eqn. 3.37 and proposed ROM in Eqn. 3.40 is shown in Fig. 7. Moreover, the information of the step response is included in Table 3.

Fig. 7. Step response of original system (Eqn. 3.35), ROM as in Eqns. 3.36 - 3.37 and proposed ROM using differentiation equation method as in Eqn. 3.40

3.4 Example-4

Considering a single-input single-output transfer function of 6*th* order system as shown [in Eq](#page-9-2)n. 3.41 [31].

$$
G(z) = \frac{0.3277z^6 + 0.9195z^5 + 1.038z^4 + 0.5962z^3 + 0.1618z^2 + 0.00698z - 0.005308}{z^6 + 1.129z^5 + 0.2889z^4 - 0.08251z^3 - 0.04444z^2 - 0.00476z} \tag{3.41}
$$

[3.4](#page-16-0).1 ROM using proposed method

The discrete system in Eqn. 3.41 has a pole at origin, therefore, the zero-order hold (ZOH) command for discrete to continuous cannot be used. The Tustin transformation is considered to transform $G(z)$ to $G(s)$ with sampling time 0.5s and is shown in Eqn. 3.42.

$$
G_6(s) = \frac{1.98s^5 + 62.53s^4 + 996.1s^3 + 8183s^2 + 3.738 \times 10^4s + 6.152 \times 10^4}{s^6 + 42.06s^5 + 701.4s^4 + 5893s^3 + 2.604 \times 10^4s^2 + 5.658 \times 10^4s + 4.619 \times 10^4}
$$
 (3.42)

Applying the differentiation method, the 3rd and 2nd order [reduc](#page-10-1)ed models are shown in Eqn. 3.43 and Eqn. 3.44, respectively.

$$
G_3(s) = \frac{1.389s^2 + 12.69s + 26.1}{s^3 + 8.838s^2 + 24s + 19.59}
$$
\n(3.43)

$$
G_2(s) = \frac{2.154s + 8.861}{s^2 + 5.431s + 6.65}
$$
\n(3.44)

Converting the above equations to discrete system is given as following Eqn. 3.45 and Eqn. 3.46.

$$
G_3(z) = \frac{0.3087z^3 + 0.3329z^2 + 0.0166z - 0.007613}{z^3 - 0.5565z^2 + 0.04164z + 0.00312}
$$
(3.45)

$$
G_2(z) = \frac{0.3938z^2 + 0.3994z + 0.005519}{z^2 - 0.4214z + 0.02087}
$$
(3.46)

System	Rise Time (s)	Settling Time (s)	Peak	Peak Time (s)
Original (Eqn. 3.11)	2.2602	3.9307	0.9991	6.9770
Prop. (Eqn. 3.34)	1.5443	2.7329	0.9996	5.3839
SE (Eqn. 3.12)	2.1889	3.2202	1.0122	4.9662
Mixed (Eqn. 3.13)	2.3585	4.1160	0.9995	7.6795
PSO (Eqn. 3.14)	2.2687	3.9175	0.9994	7.2078
GA (Eqn. 3.15)	2.2696	3.7974	1.0000	7.8662
GA (Eqn. 3.16)	2.2616	3.8443	1.0000	9.6778
Conv. (Eqn. 3.17)	2.3875	4.2478	0.9996	8.4934
BBBC (Eqn. 3.18)	2.5288	4.5538	0.9998	9.9334
BBBC (Eqn. 3.19)	2.2716	3.3278	1.0132	5.1430
PSO-Eign. (Eqn. 3.20)	2.3010	3.4111	1.0107	5.2541
FDA (Eqn. 3.21)	2.2694	4.0271	0.9995	7.8163
Error (Eqn. 3.22)	2.3413	4.0916	0.9995	7.8163
FDA (Eqn. 3.23)	2.3199	4.0643	0.9995	7.8163
Itera. $(Eqn. 3.24)$	2.2626	3.9618	0.9995	7.5639
RA (Eqn. 3.25)	2.2617	3.8446	1.0000	9.6779
FDA-Eig. (Eqn. 3.26)	2.2645	4.0177	0.9996	7.8163
PC (Eqn. 3.27)	2.3413	4.0915	0.9995	7.8163
Pade-PC (Eqn. 3.28)	1.8370	3.3411	0.9991	5.7019
Clust-ISE (Eqn. 3.29)	2.2991	4.0453	0.9995	7.8163
ISE (Eqn. 3.30)	2.2550	3.5767	1.0030	5.8832
ISE (Eqn. 3.31)	2.2617	3.8449	1.0000	9.6708

Table 2. Step response information of original system, ROM systems in literature and proposed ROM of system in Example-2

Table 3[. Ste](#page-6-6)[p res](#page-6-1)ponse information of original system, ROM systems in literature and proposed ROM of system in Example-3

System	Rise Time (s)	Settling Time (s)	Peak	Peak Time (s)
Original $(Eqn. 3.35)$	1.5621	9.0437	0.9737	2.8939
Prop. $(Eqn. 3.40)$	3.1040	6.3737	0.9557	12.4707
LS -Pade (Eqn. 3.36)	27.2242	48.8617	0.4955	86.0420
Pade (Eqn. 3.37)	3.1301	9.8472	0.9543	18.6056

3.4.2 RO[M by](#page-8-0) method in literature

The third order reduced model of above system in [31] are shown in Eqn. 3.47 and Eqn. 3.48.

$$
G_3(s) = \frac{s^2 - 0.328s + 33.22}{s^3 + 10.22s^2 + 30.55s + 24.93}
$$
\n(3.47)

$$
G_3(z) = \frac{0.1279z^3 + 0.2198z^2 + 0.2268z + 0.1349}{z^3 - 0.4231z^2 - 0.05053z + 0.006006}
$$
(3.48)

3.4.3 Si[mul](#page-10-3)ation results

The original syste[m \(E](#page-9-3)qn. 3.42), reduced models using differentiation method (Eqns. 3.43-3.44) and as proposed in Eqn. 3.47 by [31] are subjected to step response and shown in Fig. 8.

Fig. 8. Step response continuous of original system (Eqn. 3.42), ROM as in Eqn. 3.47 and proposed ROM using differentiation equation method as in Eqns. 3.43-3.44

The step response for discrete models is shown as in Fig. 9.

It is clear from both Fig. 8 and Fig. 9, that the obtained reduced ord[er mo](#page-10-1)dels with differentiation [meth](#page-11-0)od are capable to preserve the stability of the original system. The detail of st[ep res](#page-10-2)[ponse](#page-10-3) parameters is enlisted in Table 4.

Fig. 9. Step response of discrete original system (Eqn. 3.41), ROM as in Eqn. 3.48 and proposed ROM using differentiation equation method as in Eqns. 3.45-3.46

4 Conclusion

The higher order system are reduced by differentiation method and the performance is compared with other reduction methods in literature. The rise time and settling time of reduced models using proposed method are more appreciable as compared to the ROMs presented in literature. The step response of system reduced by differentiation method, as compared to other MOR methods in literature is effectively nearer to the original system response and reported with improved stability condition of the system. The application of differentiation is extended to discrete systems and found as satisfactory as in the case of continuous one. Furthermore, it have been proven to be viable to retain the stability of the system.

Competing Interests

The authors declare that no competing interests exist.

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