Concept of Free Acoustic Field

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Abstract

Aims/ objectives: To develop the concept of the acoustic field and to give the practical recommendations of its using.

Study design: The theory of functions of complex variable and the developed Dirac quantisation method were used.

Place and Duration of Study: Heat-Mass Transfer Institute of National Academy of Sciences of RB, M.V.Lomonosov Moscow State University, between November 2013 and January 2014.

Methodology: We have used generalised Cauchy-Riemann conditions for the case of a mixed vector-scalar argument in functions of complex variable and the earlier developed Dirac quantisation method.

Results: The equations of the free acoustic field are derived. They coincide in the mathematical form with the first two Maxwell equations for the free electromagnetic (EM) field. It is accentuated, that the equations in the mathematical form of the first two Maxwell equations for the free EM-field are universal equations of the Nature. They describe an any free complex-vector physical field, vector-functions of which are analytical functions. In the case of FA-field, it seems to be the strong indication taking into account the quantum Fermi liquid model of EM-field leading to the existence of phonons, accompanying the process of a photon formation, that FA-field and free EM-field are the display of the single united field, both the components of which can propagate simultaneously [however, with different velocities], for instance, in weakly absorbing media.

Conclusion: The theory proposed can be the theoretical basis for the practical realization of high temperature [room temperature and higher] superconducting states in the materials with the strong interaction of electrons with acoustic field phonons and with radiofrequency photons in magnetic resonance conditions.

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1 Introduction

Corpuscular-wave dualism is referred to fundamental concepts of the modern physics. The given concept is especially clear displayed in the crystals, where there are many fields, displaying both the aspects - wave and corpuscular ones. Quanta of the energy and the quasiimpulse of given fields have obtained the own names. Like that how the term photon describes the corpuscular aspect of an electromagnetic field in the vacuum, the terms phonon, magnon, polaron, exciton describe some quantised fields in the crystals. Let us remark that the notion of phonon was introduced in the physics of the crystals by Tamm I.E just by the above indicated analogy with the photon. The phonons are considered to be bounded up with elastic excitations, in particular, the acoustic phonons correspond to usual elastic waves. The quasiimpulse and the energy of phonon are $\vec{q} = \hbar\vec{k}$ and $E = \hbar\omega$ respectively, where $\vec{k}$ and ω are the quasiwave vector and the circular frequency of the corresponding wave of the normal lattice vibrations. The vibration state of the crystal in the harmonic approximation can be represented in the form of an ideal phonon gas.

Let us remark that the concept of the phonons is applicable also to amorphous solids including glasses for longwave acoustic vibrations, which are weak sensitive to an atomic discrete arrangement and allow the continual description of solids within the frames of the elasticity theory. The concept of the phonons is applicable to the description of quantum liquids too, at that, in the given case the phonons represent themselves the genuine particles (instead of quasiparticles in solids). The given result was used in the zeroth rest mass boson model of an EM-field in (1) and in the recent work (2), in which an EM-field is considered to be a quantum liquid, containing the phonon sybsystem. In fact, in the work (2) the structure of the EM-field vacuum was presented. It consist of massless field spin 1 bosons for the case of EM-field, photons in which are excitations - chargeless spin 1/2 topological solitons of Su-Schrieffer-Heeger family. They were represented to be the result of the spin-charge separation effect in the quantum Fermi liquid and it makes substantially more clear the nature of the corpuscular-wave dualism. Really, like to a matter atomic structure, the quantized EM-field represents itself in the model proposed the discrete "boson-atomic" structure, the individual bosons in which produce the lattice like to a genuine atomic lattice structure in condensed matter substances. The main difference between the EM-field "atomic" lattice structure and an atomic lattice structure in condensed matter solids consists in that the "atoms" in EM-field structure have zeroth rest mass. The origin of waves in a given structure is determined by the mechanism, being to be quite analogous to the Bloch waves' formation in the solid state of the condensed matter. They are harmonic trigonometric functions for the Maxwellian EM-field. At the same time, there are simultaneously the corpuscles, propagating along the given EM-field "boson-atomic" 1D-lattice structure, that is, chargeless spin 1/2 topological relativistic solitons - photons, formed in usual conditions (or spinless charged solitons in so-called "doped" EM-field structure). Within the frames of the concept proposed it is reasonable to suggest, that the way for the photon propagation [that is EM-field vacuum structure, proposed to be consisting of rest masslees bosons] was built just after of the Sun emergence (and other cosmic light sources) and the velocity of the given process seems to be unknown. The significant consequence of the given model is that the number of massles bosons in the vacuum state is finite [being to be the consequence of the finite length to a light source], that can be used for the development of an electrodynamic theory.

Let us remark, that the model proposed in (1) and in (2) is in fact the development of an idea of multiphoton "molecules". The idea of multiphoton "molecules" arose around 1910, after Debye (3) presented a simple, ondulatory method to obtain the radiation law. Debye idea was developed by using the description of "multiphoton states" in multiphoton "molecules", in (4), (5), (6): light of definite direction and frequency presents itself in units "molecules" of 0, 1, 2, ..., n, ...photons, with energy $nh\nu$, that is, with zero binding energy, and they contribute independently to the energy density. The authors of (6) accentuate, that the rationale for given assumption is, of course, in the spirit of the atomistic Democritean viewpoint, that all the photons of same frequency are created equal (identical).

Let us also remark, that the existence of phonons in weakly interacting boson gas media, which

can be considered from a quantum liquid viewpoint [that is, in the media, which mathematically are like to the boson vacuum medium used in the boson model of EM-field in (2)] is theoretically strictly proved, see, for instance, (7).

It is substantial for the presented work, that the phonon field in quantum liquids can be considered being to be an independent quantised field. On the other hand, it is well known that any quantised field is a radiation field and it can exist without the sources of its emergence. It means that the free acoustic (FA) field can exist. It can propagate in various media. The given concept was not developed earlier.

The aim of the given work is to develop the concept of the FA-field and to give the practical recommendations of its using.

2 Results and Discussion

2.1 Classical Acoustic Field

Mathematical characteristics of a classical (that is, not quantised) FA-field in the general case can be deduced from the analysis of the dynamics of a crystal lattice. It is known that the collective movement of atoms in a crystal lattice can be described by the following matrix equation

$$m \parallel \frac{d^2 u(\vec{r}_n, t)}{dt^2} \parallel = \sum_{n'=1}^{N} \parallel \alpha(\vec{r}_n - \vec{r}_n{}') \parallel \parallel u(\vec{r}_n{}', t) \parallel, \tag{2.1}$$

where $\vec{r}_n$ is the radius-vector of the nth lattice atomic site, m is the mass of an individual atom, $u(\vec{r}_n, t)$ is the displacement of the nth lattice atom from the equilibrium position, $\parallel \alpha(\vec{r}_n - \vec{r}_n{}') \parallel$ is the crystal dynamic matrix, N is the number of the atoms in a crystal. The solution of (2.1) is

$$\parallel u_s(\vec{r}_n, t) \parallel = \vec{e}_s \parallel \exp(i\vec{k}_n \vec{r}_n - i\omega_s t) \parallel, \tag{2.2}$$

where $\vec{e}_s$ is the unit vector of the polarization, determining the direction of the movement of an individual atom, $\vec{k}_n$ is the quasiwave vector, ω_s is the circular frequency, which is independent on the atom number, $s \in \overline{1, 3\nu}$, ν is the number of the atoms in an elementary unit of a crystal. It is seen from (2.2) that the acoustic classical field in crystals is characterised by the complex vector-function. Consequently, in general case, the free acoustic classical field, propagating in a medium, in which the absorption can be neglected, can be described by the complex vector-function. The mathematical structure of a complex vector-function, describing a free acoustic field has to be similar to the structure of the complex vector-function, which describe the free electromagnetic field. It follows from the affinity of an acoustic field and an electromagnetic field, established in (1) and in (2). Therefore, the free acoustic classical field can be represented in the general case in the form

$$\vec{U}(\vec{r}, t) = \vec{U}_1(\vec{r}, t) + i\vec{U}_2(\vec{r}, t), \tag{2.3}$$

where $\vec{U}_1(\vec{r}, t)$ and $\vec{U}_2(\vec{r}, t)$ are

$$\vec{U}_1(\vec{r}, t) = \sum_{\vec{k}} \vec{e}_{\vec{k}} \mathcal{U}_1 \alpha_{\vec{k}} \exp(-i\omega_{\vec{k}} t + i\vec{k}\vec{r}) + c.c.,$$

$$\vec{U}_2(\vec{r}, t) = \sum_{\vec{k}} [\vec{k} \times \vec{e}_{\vec{k}}] \mathcal{U}_2 \alpha_{\vec{k}} \exp(-i\omega_{\vec{k}} t + i\vec{k}\vec{r}) + c.c., \tag{2.4}$$

where the summation on the wave vector $\vec{k}$ is the designation of the summation on infinite sets of wave vector components, $\{k_m\}$, $m = 1, 2, 3$, $\{k_m\} \in \aleph_0$, $\mathcal{U}_1$, $\mathcal{U}_2$ are the values with the dimensions of the strengths of the components $\vec{U}_1(\vec{r}, t)$ and $\vec{U}_2(\vec{r}, t)$, $\alpha_{\vec{k}}$ is a non-dimensional amplitude, $\vec{e}_{\vec{k}}$ is the polarisation vector.

Let us find the motion equations for the given field. It can be done in a rather simple way, if to take into account that for a free acoustic field the function $\vec{U}(\vec{r}, t)$ is the analytic function. It follows immediately from the relation (2.12) for the simple case of an one mode field, if to take into account that both arguments $\vec{r} = |\vec{r}| \frac{\vec{k}}{|\vec{k}|}$ and $t = ct'$ have the same dimension along the propagation direction of $\vec{k}$-mode and to choose $\vec{e}_{\vec{k}}$ direction, coinciding, for instance, with $\vec{e_2}$-direction. Then, taking into consideration Cauchy-Riemann conditions, generalised for the case considered [that is, for the case of the mixed vector-scalar argument of functions of a complex variable], we obtain the following relationships

$$\frac{\partial \vec{U}_1(\vec{r}, t)}{\partial \vec{r}} = \frac{\partial \vec{U}_2(\vec{r}, t)}{\partial t}, \tag{2.5}$$

$$\frac{\partial \vec{U}_1(\vec{r}, t)}{\partial t} = -\frac{\partial \vec{U}_2(\vec{r}, t)}{\partial \vec{r}}, \tag{2.6}$$

where t = c t', t' is a time, c is a field propagation velocity. Calculating $\frac{\partial \vec{U}_1(\vec{r}, t)}{\partial t}$, we have the relation

$$\frac{\partial \vec{U}_1(\vec{r}, t)}{\partial \vec{r}} =$$
$$[(\frac{\partial}{\partial x}\vec{e}_1 + \frac{\partial}{\partial y}\vec{e}_2 + \frac{\partial}{\partial z}\vec{e}_3) \times$$
$$(U_{1x}(\vec{r}, t)\vec{e}_1 + U_{1y}(\vec{r}, t)\vec{e}_2 + U_{1z}(\vec{r}, t)\vec{e}_3)] \tag{2.7}$$
$$= -[\nabla \times \vec{U}_1(\vec{r}, t)]$$

Hence

$$[\nabla \times \vec{U}_1(\vec{r}, t)] = -\frac{\partial \vec{U}_2(\vec{r}, t)}{\partial t}, \tag{2.8}$$

The analogous calculation for $\frac{\partial \vec{U}_2(\vec{r}, t)}{\partial t}$ leads to the relation

$$[\nabla \times \vec{U}_2(\vec{r}, t)] = \frac{\partial \vec{U}_1(\vec{r}, t)}{\partial t} \tag{2.9}$$

Therefore, it is seen, that the equations (2.8), (2.9) in their mathematical form are coinciding with the two first Maxwell equations. They were obtained from the general conditions, mathematical expressions of which are appplicable to a very broad class of complex vector-functions - to analytical vector-functions. Consequently, they can describe any physical field, wich is described by analytical complex vector-functions.

Therefore, the equations, which in their mathematical form are coinciding with the equations (2.8), (2.9) are the universal equations of the Nature, allowing to describe any physical complex vector field, vector-functions of which are analytical vector-functions.

In the case of FA-field considered, it is reasonable to suggest that EM-field in acoustic media is the part of the joint electromagnetic-acoustic field. In other words, speaking in lapidary style, usually well detected EM-field seems to be the only a top of an iceberg. Given suggestion is based along with the coincidence of the field equations' form on the concept of the zeroth rest mass boson model of EM-field (1), (2), where the EM-field is considered to be the quantum liquid, in which the phonon sybsystem is presenting.

Many properties of FA-field and free EM-field seem to be identical. In particlar, the dual transformations for FA-field, which are like to Rainich dual transformations for EM-field (8), will take place. Taking into account the results of the paper (9), they can be represented in the form

$$\begin{bmatrix} \vec{U}'_1(\vec{r}, t) \\ \vec{U}'_2(\vec{r}, t) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \vec{U}_1(\vec{r}, t) \\ \vec{U}_2(\vec{r}, t) \end{bmatrix}, \tag{2.10}$$

where θ is the parameter, the values of which belongs to the segment $[0, 2\pi]$. The relation (2.10) can be rewritten

$$\left[\begin{array}{c} \vec{U}_1'(\vec{r},t) \\ \vec{U}_2'(\vec{r},t) \end{array} \right] = e^{-i\theta} \left[\begin{array}{c} \vec{U}_1(\vec{r},t) \\ \vec{U}_2(\vec{r},t) \end{array} \right], \tag{2.11}$$

It was taken into consideration, that to any matrix, which has the structure, given by the $[2 \times 2] matrix in the rights side of the relati$

$$a + ib \rightarrow \left[\begin{array}{cc} a & -b \\ b & a \end{array} \right], (2.12) \text{determined by the left side of the relation (2.10). It is the consequence}$$

of the bijectivity of mapping (2.12). In other words, to the real plane determined by orthogonal vectors $\vec{U}_1(\vec{r},t)$, $\vec{U}_2(\vec{r},t)$ is setting up in the correspondence the complex plane determined by vectors $\vec{U}_1(\vec{r},t)$, $i\vec{U}_2(\vec{r},t)$. It is evident, that the vectors $\vec{U}_1'(\vec{r},t)$ and $\vec{U}_2'(\vec{r},t)$ will consist both of even and uneven components under space inversion transformations. So, it is evident from the relation (2.11), that if one component of, for instance, $\vec{U}_1'(\vec{r},t)$ will be uneven under reflection in the plane, situated transversely to the abscissa-axis, then the second component will be even.

Invariants of FA-field if $\theta \neq 0$ are the following

$$\left[\vec{U}_1^2(\vec{r},t) - \vec{U}_2^2(\vec{r},t) \right] e^{-2i\theta}$$
$$+ 2i \left[\vec{U}_1(\vec{r},t)\vec{U}_2(\vec{r},t)(\vec{r},t) \right] e^{-2i\theta} = inv, \tag{2.13}$$

that is,

$$(\vec{U}_1^2(\vec{r},t) - \vec{U}_2^2(\vec{r},t)) \cos 2\theta +$$
$$2(\vec{U}_1(\vec{r},t)\vec{U}_2(\vec{r},t)) \sin 2\theta = I_1' = inv$$
$$2(\vec{U}_1(\vec{r},t)\vec{U}_2(\vec{r},t)) \cos 2\theta -$$
$$(\vec{U}_1^2(\vec{r},t) - \vec{U}_2^2(\vec{r},t)) \sin 2\theta = I_2' = inv. \tag{2.14}$$

In the case of $\theta = 0$ the FA-field invariants have the form, mathematically coinciding with the well known form for the invariants of the single charge electrodynamics. They are

$$(\vec{U}_1^2(\vec{r},t) - \vec{U}_2^2(\vec{r},t)) = I_1 = inv$$
$$(\vec{U}_1(\vec{r},t)\vec{U}_2(\vec{r},t)) = I_2 = inv. \tag{2.15}$$

2.2 Classical and Quantized Cavity Acoustic Field

Suppose an acoustic field in a volume rectangular cavity, which made up of perfect walls. Suppose also, that the field is linearly polarized and without any restriction of the commonness let us choose the one of three possible polarization (the case $\nu = 1$ is considered) of the FA-field component $\vec{U}_1(\vec{r},t)$ along x-direction. Then the vector-function $\vec{U}_1(\vec{r},t)$ can be represented in the form of Fourier sine series

$$U_{1x}(z,t)\vec{e}_x = \left[\sum_{\alpha=1}^{\infty} A_\alpha^{U_1} q_\alpha(t) \sin(k_\alpha z) \right] \vec{e}_x, \tag{2.16}$$

where

$$k_\alpha = \alpha\pi/L; \alpha \in N, \tag{2.17}$$

$q_\alpha(t)$ is the amplitude of α-th normal mode in the cavity, $A_\alpha^E = \sqrt{2\omega_\alpha^2 m_\alpha/V}$, $\omega_\alpha = \alpha\pi c/L$, L is the cavity length along z-axis, c is sound velocity, V is the cavity volume, m_α is the parameter, which is introduced to obtain the analogy with a lattice harmonic oscillator. Let us remember, that the expansion in Fourier series instead of Fourier integral expansion is determined by a discontinuity of $\vec{k}$-space, which is the result of finiteness of cavity volume. Particular sine case of Fourier series is the consequence of boundary conditions

$$[\vec{n} \times \vec{U}_1(\vec{r}, t)]|_S = 0, (\vec{n}\vec{U}_2(\vec{r}, t))|_S = 0, \tag{2.18}$$

which are held true for the perfect cavity considered. Here $\vec{n}$ is the normal to the surface S of the cavity. It can be shown, that $\vec{U}_1(\vec{r}, t)$ represents itself a standing wave along z-direction. For $\vec{U}_2(\vec{r}, t)$ component, taking into consideration the equations (2.8), (2.9), we have the expression

$$\vec{U}_2(\vec{r}, t) = \left[\sum_{\alpha=1}^{\infty} A_\alpha^{U_2} \frac{1}{k_\alpha} \frac{dq_\alpha(t)}{dt} \cos(k_\alpha z) + f_\alpha(t)\right] \vec{e}_y, \tag{2.19}$$

The partial solution, in which the functions $\{f_\alpha(t)\}$ are identically zero is

$$\vec{U}_2^{[1]}(\vec{r}, t) = \left[\sum_{\alpha=1}^{\infty} A_\alpha^{U_1} \frac{1}{k_\alpha} \frac{dq_\alpha(t)}{dt} \cos(k_\alpha z)\right] \vec{e}_y. \tag{2.20}$$

At the same time, there is the second physically substantial solution of the equations (2.8), (2.9). Really, from the expression (2.19) for the field component $\vec{U}_2(\vec{r}, t)$ differential equations for $\{f_\alpha(t)\}$, $\alpha \in N$, can be obtained. They are

$$\frac{df_\alpha(t)}{dt} + A_\alpha^{U_1} \frac{1}{k_\alpha} \frac{\partial^2 q_\alpha(t)}{\partial t^2} \cos(k_\alpha z)$$
$$- A_\alpha^{U_1} k_\alpha q_\alpha(t) \cos(k_\alpha z) = 0. \tag{2.21}$$

The solutions of (2.21) are

$$f_\alpha(t) = A_\alpha^{U_1} \cos(k_\alpha z) \left[k_\alpha \int_0^t q_\alpha(\tau)d\tau - \frac{dq_\alpha(t)}{dt} \frac{1}{k_\alpha}\right]. \tag{2.22}$$

Then $\vec{U}_2(\vec{r}, t)$ component can be represented in the form

$$\vec{U}_2^{[2]}(\vec{r}, t) = -\left\{\sum_{\alpha=1}^{\infty} A_\alpha^{U_2} q_\alpha'(t) \cos(k_\alpha z)\right\} \vec{e}_y, \tag{2.23}$$

where $A_\alpha^{U_2} = \sqrt{2\omega_\alpha^2 m_\alpha/V}$. The similar consideration gives the second solution for $\vec{U}_1(\vec{r}, t)$ FA-field component

$$\vec{U}_1^{[2]}(\vec{r}, t) = \left\{\sum_{\alpha=1}^{\infty} A_\alpha^{U_1} q_\alpha''(t) \sin(k_\alpha z)\right\} \vec{e}_x. \tag{2.24}$$

The functions $q_\alpha'(t)$ and $q_\alpha''(t)$ in (2.23) and (2.24) are

$$q_\alpha'(t) = \omega_\alpha \int_0^t q_\alpha(\tau)d\tau$$
$$q_\alpha''(t) = \omega_\alpha \int_0^t q_\alpha'(\tau')d\tau' \tag{2.25}$$

The class of field functions $\{q_\alpha(t)\}$ has to satisfy to the equations (2.8), (2.9). The given condition allows to find the corresponding class in the explicit form. The functions of the given clas will satisfy to the differential equations

$$\frac{d^2 q_\alpha(t)}{dt^2} + k_\alpha^2 q_\alpha(t) = 0, \alpha \in N. \tag{2.26}$$

Consequently, we have

$$q_\alpha(t) = C_{1\alpha}e^{i\omega_\alpha t} + C_{2\alpha}e^{-i\omega_\alpha t}, \alpha \in N, \tag{2.27}$$

where $C_{1\alpha}, C_{2\alpha}, \alpha \in N$, are arbitrary constants. Thus, the solutions are complex-valued functions, although the variables in free acoustic field differential equations are real-valued vector-functions. The given result is well known in the theory of differential equations. It means, that generally the field functions for the free acoustic field in the cavity produce the complex space. On the other hand, the equation (2.26) has also the only real-valued general solution, which can be represented in the form

$$q_\alpha(t) = B_\alpha \cos(\omega_\alpha t + \phi_\alpha), \tag{2.28}$$

where $B_\alpha, \phi_\alpha, \alpha \in N$, are arbitrary constants.

So, for the classical cavity FA-field case we can in the principle restrict themselves to the only real-valued general solution of FA-field equations.

Owing to the fact, that the solutions of FA-field equations have the simple form of harmonic trigonometrical functions, it is easily to establish that the second solution for acoustic field functions differs from the first solution the only by sign, that is substantial, and by inessential integration constants. Integration constants can be taken into account by means of the redefinition of the factor m_α in field amplitudes.

Let us briefly consider the quantization of the free acoustic field. We begin the consideration like to the canonical quantization, from the classical Hamiltonian, which for the first partial classical solution of field equations is

$$\mathcal{H}(t) = \frac{1}{2}\int\limits_{(V)} \left[\vec{U}_1^2(z,t) + \vec{U}_2^2(z,t)\right] dxdydz$$

$$= \frac{1}{2}\sum_{\alpha=1}^{\infty}\left[m_\alpha\omega_\alpha^2 q_\alpha^2(t) + \frac{p_\alpha^2(t)}{m_\alpha}\right], \tag{2.29}$$

where

$$p_\alpha = m_\alpha\frac{dq_\alpha(t)}{dt}. \tag{2.30}$$

We set up then in the correspondence to canonical variables $q_\alpha(t), p_\alpha(t)$, determined by the first partial solution of field equations, the operators by an usual way

$$[\hat{p}_\alpha(t), \hat{q}_\beta(t)] = i\hbar\delta_{\alpha\beta}$$
$$[\hat{q}_\alpha(t), \hat{q}_\beta(t)] = [\hat{p}_\alpha(t), \hat{p}_\beta(t)] = 0, \tag{2.31}$$

where $\alpha, \beta \in N$. Let us define the operator functions of the time $\hat{a}_\alpha(t)$ and $\hat{a}_\alpha^+(t)$

$$\hat{a}_\alpha(t) = \frac{1}{\sqrt{2\hbar m_\alpha\omega_\alpha}}\left[m_\alpha\omega_\alpha\hat{q}_\alpha(t) + i\hat{p}_\alpha(t)\right]$$

$$\hat{a}_\alpha^+(t) = \frac{1}{\sqrt{2\hbar m_\alpha\omega_\alpha}}\left[m_\alpha\omega_\alpha\hat{q}_\alpha(t) - i\hat{p}_\alpha(t)\right]. \tag{2.32}$$

Then the operator functions of canonical variables can be represented in the form

$$\hat{q}_\alpha(t) = \sqrt{\frac{\hbar}{2m_\alpha\omega_\alpha}}\left[\hat{a}_\alpha^+(t) + \hat{a}_\alpha(t)\right]$$

$$\hat{p}_\alpha(t) = i\sqrt{\frac{\hbar m_\alpha\omega_\alpha}{2}}\left[\hat{a}_\alpha^+(t) - \hat{a}_\alpha(t)\right]. \tag{2.33}$$

FA-field operator functions are obtained taking into account (2.33) right away and they are

$$\hat{\vec{U}}_1(\vec{r},t) = \{\sum_{\alpha=1}^{\infty}\sqrt{\frac{\hbar\omega_\alpha}{V}}\left[\hat{a}_\alpha^+(t) + \hat{a}_\alpha(t)\right]\sin(k_\alpha z)\}\vec{e}_x, \tag{2.34}$$

and

$$\hat{\vec{U}}_2(\vec{r},t) = i\{\sum_{\alpha=1}^{\infty} \sqrt{\frac{\hbar\omega_\alpha}{V}}\left[\hat{a}_\alpha^+(t) - \hat{a}_\alpha(t)\right]\cos(k_\alpha z)\}\vec{e}_y,$$ (2.35)

Taking into account the relationships (2.34), (2.35) and FA-field equations (2.8), (2.9), we find an explicit form for the dependencies of operator functions $\hat{a}_\alpha(t)$ and $\hat{a}_\alpha^+(t)$ on the time. They are the following

$$\hat{a}_\alpha^+(t) = \hat{a}_\alpha^+(t=0)e^{i\omega_\alpha t},$$
$$\hat{a}_\alpha(t) = \hat{a}_\alpha(t=0)e^{-i\omega_\alpha t},$$ (2.36)

where $\hat{a}_\alpha^+(t=0), \hat{a}_\alpha(t=0)$ are constant operators. Physical sense of operator time dependent functions $\hat{a}_\alpha^+(t)$ and $\hat{a}_\alpha(t)$ is understandable. They are creation and annihilation operators of the α-mode phonon in the cavity. They are continuously differentiable operator functions of a time. It means, that the time of phonon creation (or annihilation) can be determined strictly, at the same time operator functions $\hat{a}_\alpha^+(t)$ and $\hat{a}_\alpha(t)$ do not curry any information on the place, that is on space coordinates of a given event.

Thus, we have considered briefly the local time quantization of a free acoustic field. The local space quantization and the local space-time quantization of a free acoustic field can be made in the way, which was developed for the local space quantization and for the local space-time quantization of a cavity EM-field in (9). The possibility to quantize the FA-field means that the FA-field is really the radiation field and it can exist without the sources of its initial emergence.

It seems to be also essential, that complex exponential dependencies in (2.36) cannot be replaced by the real-valued harmonic trigonometrical functions. The proof is identical to the proof of an analogous statement for Maxwellian EM-field (9). Consequently, the quantized free acoustic field differs qualitatively from the classical acoustic field.

Practical significance of the results above presented is the following. By the study of the properties of the solids or quantum liquids [for the subsequent using in engeneering] situated in the cavity [for instance, by radio or optical spectroscopy methods] we have to take into account that two discrete $\vec{k}$ spaces will be formed with different discreteness extent, determined by an interatomic distance in solids on the one hand and by cavity modes on the second hand. At that, they can be incommensurable. In given case, the cavity FA-field phonons can lead by the interaction with an electronic subsystem of solids practically the only to the exchange by the energy and the impulse between electrons [that is, to the formation of Cooper pairs]. Really, the processes of electron scattering with the generation or the absorption of FA-field phonons are impossible, since the probability of the processes of electron scattering with the generation or the absorption of phonons, for instance, for nondegenerated semiconductors is determined by the known expression

$$W_{\vec{k}_l \to \vec{k}_m}^{\pm \vec{q}_j} = \frac{2\pi}{\hbar}|\,M_{\vec{k}_l \to \vec{k}_m}^{\pm \vec{q}_j}\,|^2 \delta(\mathcal{E}_{\vec{k}_l} - \mathcal{E}_{\vec{k}_m} \mp \hbar\omega_{\vec{q}_j})[N_{\vec{q}_j} + \frac{1}{2} \pm \frac{1}{2}],$$ (2.37)

upper signs in which correspond to the phonon emission, lower signs correspond to the phonon absorption. Here $\vec{k}_l$, $\vec{k}_m$ are the quasiwave vectors of the lattice, $l, m = 1, 2, ..., N'$, determined by discrete structure of $\vec{k}$ lattice space with the discreteness

$$k_n = \frac{2\pi n}{N'}, n = 1, 2, ..., N',$$ (2.38)

N' is the number of the atoms in the lattice, $\mathcal{E}_{\vec{k}_l}$ and $\mathcal{E}_{\vec{k}_m}$ are the values of the energy of the electron in the states with quasiwave vectors $\vec{k}_l$ and $\vec{k}_m$ correspondingly, $\vec{q}_j$ is quasiwave vector of absorbed or emitted phonon of the lattice with the value q_j, satisfying to the relation (2.38), by the replacement $k \to q$, $\omega_{\vec{q}_j}$ is circular phonon frequency, $N_{\vec{q}_j}$ is a number of phonons with quasiwave vector $\vec{q}_j$, $M_{\vec{k}_l \to \vec{k}_m}^{\pm \vec{q}_j}$

is matrix element of the transition $\vec{k}_l \rightarrow \vec{k}_m$, for which the conservation law of the quasiimpulse, satysfying to the relation

$$\hbar\vec{k}_l - \hbar\vec{k}_m \mp \hbar\vec{q}_j = \vec{b} \tag{2.39}$$

takes place. Here $\vec{b}$ is an arbitrary vector of a reciprocal lattice. Let us remember, that the thansitions with $\vec{b} = 0$ are normal transitions, if $\vec{b} \neq 0$, the thansitions are accompanying with Peierls transfer processes. It can be concluded, that FA-field phonons will be not satisfy in the general case to the expression (2.37) and consequently they will not contribute to scattering processes. It follows from that that they will have the quasiimpulse distribution obeing to the relation (2.17) in which $\vec{k}$-space discretness is determined by the cavity length in the FA-field propagation direction. It is understandable that in the case of incommensurate cavity and lattice $\vec{k}$ space discretnesses all the FA-field phonons will have the energy and quasiimpulse characteristics, which will be not satisfying to the relation (2.37). Let us remark, that the mechanism of the creation of the system of coherent resonance phonons [that is in fact the mechanism of the generation of FA-field phonons in the terminology of the concept proposed in the given paper] is described in the case of the strong interaction of electrons with phonons, produced in the resonance conditions and with cavity photons (the term "strong" means that practically all conducting electrons in a sample are involved in the given process directly or undirectly and they will not participate in the scattering processes with equilibrium own lattice phonons, see further) in the work (11).

Simultaneously, the processes of electron scattering with the emission or the absorption of own lattice equilibrium phonons will be suppressed in the radiospectroscopy case. Really, a photon flux incoming in the cavity, for instance, in the microwave region is very intensive. So, the photon flux, corresponding to the microwave power in 100 mW, that is, to the standard power of microwave generators in ESR-spectrometers without an attenuation, is equal to $\approx 10^{22}$ photons/s of the frequency equaled to 10 GHz. It leads by a strong electron-photon interaction, realised by a spin subsystem of a sample electron system, to a very great number of cavity FA-field hypersound phonons. At the same time, it is well known, that the interaction which produces the energy difference between the normal and superconducting phases in the theory of the superconductivity arises from the virtual exchange of phonons and the screened Coulomb repulsion between electrons. Other interactions are thought to be essentially the same in both the states, their effects thus cancelling in the energy difference. The Hamiltonian for the the electron-phonon interaction, which comes from the virtual exchange of phonons between the electrons is the following (12)

$$\mathcal{H}_{e-ph} =$$
$$\hbar \sum_{\vec{k}\vec{k}'\sigma\sigma'\vec{q}} \frac{\omega_{\vec{q}}|M_{\vec{q}}|^2 \hat{c}^+(\vec{k}'-\vec{q},\sigma')\hat{c}(\vec{k}',\sigma')\hat{c}^+(\vec{k}+\vec{q},\sigma)\hat{c}(\vec{k},\sigma)}{(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}})^2 - (\hbar\omega_{\vec{q}})^2}, \tag{2.40}$$

where $\varepsilon_{\vec{k}}$ is the Bloch energy measured relative to the Fermi energy, $M_{\vec{q}}$ is matrix element for phonon-electron interaction, $\hat{c}^+(\vec{k},\sigma)$, $\hat{c}(\vec{k},\sigma)$ are fermion creation and annihilation operators, based on the renormalized Bloch states specified by quasiwave vector $\vec{k}$ and spin projection σ, which satisfy the usual Fermi commutation relations. The most significant transitions are those for which $|\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}| \ll \hbar\omega_{\vec{q}}$. It is evident, that by the generation of the coherent system of resonance hypersound phonons the given condition will be easily fulfilled. The probality of the given process is proportional to $\sum_{\vec{q}}$ in the expression (2.40), that is, to the number of FA-field hypersound phonons wich can be produced in a very large amount. It gives clear proof for the participation of the conducting electron system in the materials with the strong electron-phonon and electron-photon interactions at ESR conditions the only in the virtual exchange processes, excluding the scattering processes.

At the same time it is well known, that just the electron scattering processes are the main processes determining the electrical resistance. Thus, it is the direct way for the formation of the superconducting states in ESR conditions or another magnetic resonance conditions by room temperature

and even by more high temperatures in conductors or semiconductors in which the strong interaction of sample electrons with cavity FA-field phonons is realized. The given conclusion has the experimental confirmation. Peculiarities of spin-wave resonance observed in carbon nanotubes, produced by high-energy ion beam modification of diamond single crystals in $\langle 100 \rangle$ direction (10) allowed to insist on the formation in given nanotubes of s^+-superconductivity at the room temperature, coexisting with uncompensated antiferromagnetic ordering just in ESR conditions. The role of hypersound resonance phonons produced in ESR-spectrometer cavity, that is, the phonons which can be referred to cavity FA-field quanta, in the formation of the superconducting state in a microwave frequency range is proposed in the cited work to be crucial.

Let us consider the discontinuity extent of $\vec{k}$-space. It can be obtained from the expression $k_{\alpha,L} = \alpha\pi/L$, considered to be the function of α and L. It is the following

$$\Delta k_{\alpha,L} = \frac{\alpha\pi}{L}\Delta\alpha - \frac{\alpha\pi}{L^2}\Delta L, \tag{2.41}$$

$\Delta\alpha = 1$, ΔL is the small increase of the cavity length. Hence we see, that the discontinuity of $\vec{k}$-space is increasing by cavity length decreasing, that is, if $\Delta L = -|\Delta L|$. Since the depencence of $\Delta k_{\alpha,L}$ on L is substantional, we can expect that the quantization of EM-field will be also substantional in a microwave range, if a cavity length for the same frequency can be strongly diminished. It is possible in the case when electron-photon interaction is rather strong. In other words, it can lead to the appearance of quantum phenomena, which can be described the only within the frames of the quantum field model and cannot be described by any semiclassical model. The formation of an additional cavity by a sample itself in the measuring cavity of an ESR spectrometer can be an experimental criterion for the notion of the strong electron-photon interaction in the radiospectroscopy range.

The affinity of an EM-field and an acoustic field above indicated allows to insist that in the given case [that is, by the formation of an additional cavity by a sample itself in the measuring cavity of an ESR spectrometer] the acoustic field part, connected with an EM-field will be also quantized, leading to revealing of new quantum phenomena, connected with an acoustic field, in particular, to quantum acousic Rabi oscillations, having the time discrete structure, predicted in (11). Moreover, the given new quantum phenomenon can be observed in the samples with any atomic structures, in particular, in the samples with disordered amorphous atomic structures, where the own phonon spectral distribution is characterised by rather broad lines without a clear peak structure.

Let us remark, that the substantial coherent phonon generation, which will be occured just in the sample, is possible in magnetic resonance conditions in the case when the sample itself is the additional cavity, that, in its turn, is possible by the strong interaction of electrons with microwave photons. The strong interaction of electrons with resonance phonons will be realised then in fact automatically, since resonance phonons, frequency of which is not coinciding with the frequency of own sample phonon modes will have very long life time. In other words, the relaxation time of coherent phonons will be substantially greater in the comparison with the period of acoustic Rabi oscillations. It is the mathematical criterion for the strong electron-phonon interaction. An analogous mathematical criterion takes place for the strong electron-photon interaction.

3 Conclusions

The concept of the classical and the quantum free acoustic field is theoretically considered for the first time. The equations of the free acoustic field are derived. They coincide in the mathematical form with the two first Maxwell equations for a free electromagnetic field. It is concluded, that the equations, which in their mathematical form are coinciding with the equations (2.8), (2.9) are the universal equations of the Nature. They allow to describe any physical complex vector field, vector-functions of which are analytical vector-functions.

It is proposed, taking into account the quantum Fermi liquid model of an electromagnetic field leading to the existence of phonons, accompanying the process of a photon formation, that the same form of field equations is the strong indication, that both the free acoustic field and the free electromagnetic field are the display of the single united field, both the components of which can propagate simultaneously [however, with different velocities], for instance, in weakly absorbing media.

The theory of the free acoustic field can be the theoretical basis for the practical realization of superconducting states at high temperatures in the materials with the strong interaction of electrons with acoustic field phonons and radiofrequency-range photons in magnetic resonance conditions. The given conclusion has the experimental confirmation for the case of electron spin resonance, which is briefly described in the paper presented.

Competing interests

The authors declare that no competing interests exist.

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