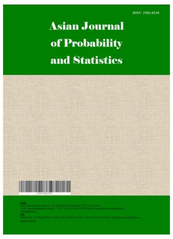

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Solving Ordinary Differential Equation of Higher Order by Adomian Decomposition Method

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Aims/ Objectives: In this article, we use Adomian Decomposition method (ADM) for solving initial value problems in the higher order ordinary differential equations. Many researchers have used the ADM in order to find convergent as well as exact solutions of different types of equations. Therefore, the ADM is considered as an effective and successful method for solving differential equations. In this paper, we presented some suggested amendments to the ADM by using a new differential operator in order to find solutions for higher order types of equations. We demonstrated the effectiveness of this method through many examples and we find out that we get an approximate solutions using the proposed amendments. We can conclude that the suggested modification of ADM is afftective and produces reliable results.

Keywords: Adomian decomposition method; initial conditions; higher order ordinary differential equation.

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1 Introduction

Adomian Decomposition Method (ADM) is a new and accomplished method for solving linear and non-linear ordinary differential equation [1,2,3]. In recent years, ADM has been used in a wide range for solving linear and non-linear equation in applied sciences for as in [4,5]. This method attracted the attention of many scientists and researchers. In [6] Yahya Qaid Hasan and Sumayah Ghaleb Othman studied non-linear oscillatory systems of higher order, and they show the accuracy of the solutions arrived at using this article is to find approximate solutions with high efficiency by using ADM. For this reason, we introduce a new differential operator which can be used in order to get reliable solutions for initial value problems in the higher ordinary differential equations.

1.1 General construction structure of equation

We suggest a new differential operator L as follows:

$$L(.) = e^{-kx}\frac{d^{(m+1)}}{dx^{(m+1)}}e^{kx}(.), \tag{1}$$

whereas $m \geq 1$, k constant, g(x,y) is given function.

If we put $m = 1$, in eq. (1), we have:

$$\begin{aligned} Ly &= e^{-kx}\frac{d^{(2)}}{dx^{(2)}}e^{kx}(y), \\ &= e^{-kx}\frac{d}{dx}[e^{kx}y' + ke^{kx}y], \\ &= e^{-kx}[e^{kx}y'' + ke^{kx}y' + ke^{kx}y' + k^2e^{kx}y], \\ &= e^{-kx}[e^{kx}y'' + 2ke^{kx}y' + k^2e^{kx}y], \\ y'' &+ 2ky' + k^2y = g(x,y), \end{aligned} \tag{2}$$

if we put $m = 2$, in eq. (1), by the same way, we obtain:

$$y''' + 3ky'' + 3k^2y' + k^3y = g(x,y), \tag{3}$$

if we put $m = 3$, in eq. (1), we obtain:

$$y'''' + 4ky''' + 6k^2y'' + 4k^3y' + k^4y = g(x,y), \tag{4}$$

if we put $m = 4$, in eq. (1), we obtain:

$$y^{(5)} + 5ky^{(4)} + 10k^2y^{(3)} + 10k^3y^{(2)} + 5k^4y' + k^5y = g(x,y), \tag{5}$$

so we continue and finally we get:

$$\sum_{n=0}^{m} k^n \binom{m}{n} y^{(m-n+1)} + \sum_{n=0}^{m} k^{n+1}\binom{m}{n} y^{(m-n)} = g(x,y). \tag{6}$$

Theorem: If m∈N then

$$e^{-kx}\frac{d^{(m+1)}}{dx^{(m+1)}}e^{kx}(y) = \sum_{n=0}^{m} k^n \binom{m}{n} y^{(m-n+1)} + \sum_{n=0}^{m} k^{n+1}\binom{m}{n} y^{(m-n)}, \tag{7}$$

Proof: We prove that by mathematical induction:

If $m = 1$, this statement is

$$e^{-kx}\frac{d^2}{dx^2}e^{kx}y = \sum_{n=0}^{1} k^n \binom{1}{n} y^{(2-n)} + \sum_{n=0}^{1} k^{n+1}\binom{1}{n} y^{(1-n)},$$

where the left-hand side is $y'' + 2ky' + k^2y$, and the right-hand side is $y'' + 2ky' + k^2y$, then equation

$$e^{-kx}\frac{d^2}{dx^2}e^{kx}y = \sum_{n=0}^{1} k^n \binom{1}{n} y^{(2-n)} + \sum_{n=0}^{1} k^{n+1}\binom{1}{n} y^{(1-n)},$$

is holds.
We must now prove $S_h \Longrightarrow S_{h+1}$. That is, we must show that if

$$e^{-kx}\frac{d^{(h+1)}}{dx^{(h+1)}}e^{kx}y = \sum_{n=0}^{h} k^n \binom{h}{n} y^{(h-n+1)} + \sum_{n=0}^{h} k^{n+1}\binom{h}{n} y^{(h-n)}.$$

Then

$$e^{-kx}\frac{d^{(h+2)}}{dx^{(h+2)}}e^{kx}y = \sum_{n=0}^{h+1} k^n \binom{h+1}{n} y^{(h-n+2)} + \sum_{n=0}^{h+1} k^{n+1}\binom{h+1}{n} y^{(h-n+1)},$$

we use direct proof, suppose

$$e^{-kx}\frac{d^{(h+1)}}{dx^{(h+1)}}e^{kx}y = \sum_{n=0}^{h} k^n \binom{h}{n} y^{(h-n+1)} + \sum_{n=0}^{h} k^{n+1}\binom{h}{n} y^{(h-n)}.$$

Then

$$\begin{aligned}
e^{-kx}\frac{d^{(h+2)}}{dx^{(h+2)}}e^{kx}y &= e^{-kx}\frac{d^{(h+1)}}{dx^{(h+1)}}(e^{kx}y' + ke^{kx}y),\\
&= e^{-kx}\frac{d^{(h+1)}}{dx^{(h+1)}}e^{kx}y' + e^{-kx}\frac{d^{(h+1)}}{dx^{(h+1)}}ke^{kx}y,\\
&= \sum_{n=0}^{h} k^n \binom{h}{n} y^{(h-n+2)} + \sum_{n=0}^{h} k^{n+1}\binom{h}{n} y^{(h+1-n)}\\
&+ \sum_{n=0}^{h} k^{n+1} \binom{h}{n} y^{(h-n+1)} + \sum_{n=0}^{h} k^{n+2}\binom{h}{n} y^{(h-n)},\\
&= y^{h+2} + \sum_{n=1}^{h} k^n \binom{h}{n} y^{(h-n+2)} + \sum_{n=0}^{h} k^{n+1}\binom{h}{n} y^{(h-n+1)} +\\
&ky^{h+1} + \sum_{n=1}^{h} k^{n+1} \binom{h}{n} y^{(h-n+1)} + \sum_{n=0}^{h} k^{n+2}\binom{h}{n} y^{(h-n)},\\
&= y^{h+2} + \sum_{n=1}^{h} k^n \binom{h}{n} y^{(h-n+2)} + \sum_{n=1}^{h} k^{n}\binom{h}{n-1} y^{(h-n+2)} +\\
&ky^{h+1} + \sum_{n=1}^{h} k^{n+1} \binom{h}{n} y^{(h-n+1)} + \sum_{n=1}^{h} k^{n+1}\binom{h}{n-1} y^{(h-n+1)},\\
&= y^{(h+2)} + \sum_{n=1}^{h} k^n [\binom{h}{n} + \binom{h}{n-1}] y^{(h-n+2)} +
\end{aligned}$$

$$ky^{h+1} + \sum_{n=1}^{h} k^{n+1}[\binom{h}{n} + \binom{h}{n-1}]y^{(h-n+1)},$$

$$= \sum_{n=0}^{h+1} k^{n} \binom{h+1}{n} y^{(h-n+2)} + \sum_{n=0}^{h+1} k^{n+1} \binom{h+1}{n} y^{(h-n+1)}.$$

Therefore

$$e^{-kx}\frac{d^{(m+1)}}{dx^{(m+1)}}e^{kx}y = \sum_{n=0}^{m} k^{n}\binom{m}{n}y^{(m-n+1)} + \sum_{n=0}^{m} k^{k+1}\binom{m}{n}y^{(m-n)}.$$

It follows by induction that

$$e^{-kx}\frac{d^{(m+1)}}{dx^{(m+1)}}e^{kx}y = \sum_{n=0}^{m} k^{n}\binom{m}{n}y^{(m-n+1)} + \sum_{n=0}^{m} k^{k+1}\binom{m}{n}y^{(m-n)}. \tag{8}$$

2 Adomian Decomposition Method

We consider the ordinary differential equation in (6):
with initial conditions

$$y(0) = c_0, y'(0) = c_1, y''(0) = c_2, ..., y^{(m)}(0) = c_i.$$

Where $i = 0, 1, 2, 3, ...$
The differential operator L for equation (6) as below:

$$Ly = g(x, y) \tag{9}$$

where

$$L(.) = e^{-kx}\frac{d^{(m+1)}}{dx^{(m+1)}}e^{kx}(.), \tag{10}$$

and the invers differential operator L^{-1} is:

$$L^{-1}(.) = e^{-kx}\underbrace{\int_0^x ... \int_0^x}_{(m+1)} e^{kx}(.)\underbrace{dx...dx}_{(m+1)}. \tag{11}$$

Applying L^{-1} on both sides of eq. (9), we have

$$y(x) = \gamma(x) + L^{-1}(g(x, y)), \tag{12}$$

where $\gamma(x)$ the part arising from using the auxiliary conditions.

By Adomian method we obtian the solution who gives in the form $y(x)$ and the function $g(x, y)$ is infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{13}$$

and

$$g(x, y) = \sum_{n=0}^{\infty} A_n, \tag{14}$$

where the component $y_n(x)$ for this solution $y(x)$ you must find it in a repeated formula. Appointed algorithms were seen in [2,7] to formulate Adomian polynomials, the below algorithm:

$$A_0 = h(y_0),$$

$$A_1 = y_1 h'(y_0),$$
$$A_2 = y_2 h'(y_0) + \frac{1}{2!} y_1^2 h''(y_0),$$
$$A_3 = y_3 h'(y_0) + y_1 y_2 h''(y_0) + \frac{1}{3!} y_1^3 h'''(y_0), \quad (15)$$
$$\ldots$$

Can be used to structure Adomian polynomilas, when $h(y)$ is a non-linear function. From (12),(13) and (14), we obtian:

$$\sum_{n=0}^{\infty} y_n(x) = \gamma(x) + L^{-1} \sum_{n=0}^{\infty} A_n. \quad (16)$$

The components $y(x)$ which we obtain by Adomian Decomposition Method, can be clarify as follows:

$$y_0 = \gamma(x),$$
$$y_{n+1} = L^{-1} A_n, n \geq 0, \quad (17)$$

which gives

$$y_1 = L^{-1} A_0,$$
$$y_2 = L^{-1} A_1,$$
$$y_3 = L^{-1} A_2, \quad (18)$$
$$\ldots$$

From (15) and (18), we can define the components $y_n(x)$, and therefore the series solution of $y(x)$ in (14) we can get it directiy.

$$\psi_n(x) = \sum_{i=0}^{n-1} yi, \quad (19)$$

it can be used to approximate the exact solution.

3 Numerical Applications

In this section, we give some examples of different order to show the accuracy and speed of this method.

Prblem 1. Take into account the following propblem: When $m = 1, k = 2$ in Eq. (6) we get:

$$y'' + 4y' + 4y = (6 + 8x + 4x^2)e^{x^2} + x^2 - \ln y, \quad (20)$$
$$y(0) = 1, y'(0) = 0,$$

with exact solution e^{x^2}. Eq.(20) we can write it as below:

$$Ly = (6 + 8x + 4x^2)e^{x^2} + x^2 - \ln y, \quad (21)$$

where the differential operator in (1) become

$$L(.) = e^{-2x} \frac{d^2}{dx^2} e^{2x}(.),$$

and

$$L^{-1}(.) = e^{-2x} \int_0^x \int_0^x e^{2x}(.) dx dx.$$

Applying the L^{-1} on Eq. (21) and using initial conditions, we have

$$y(x) = ((1 + 2x)e^{-2x} + L^{-1}(6 + 8x + 4x^2)e^{2x} - L^{-1} \ln y, \quad (22)$$

we can replace $y(x)$ in Eq (22) by $y_n(x)$ as:

$$\sum_{n=0}^{\infty} y_n(x) = ((1+2x)e^{-2x} + L^{-1}(6+8x+4x^2)e^{2x}) - L^{-1}\ln y_n,$$

where

$$y_0 = ((1+2x)e^{-2x} + L^{-1}(6+8x+4x^2)e^{2x}),$$
$$y_{n+1} = -L^{-1}A_n, n \geq 0,$$

where A_n Adomian polynomials defind by:

$$A_n = \ln y_n,$$
$$A_0 = \ln y_0,$$
$$A_1 = \frac{y_1}{y_0}$$

Then

$$y_0 = 1 + x^2 + \frac{7\,x^4}{12} - \frac{x^5}{15} + \frac{x^6}{5} - \frac{4\,x^7}{315} + \frac{23\,x^8}{504} - \frac{x^9}{945} + \frac{139\,x^{10}}{16200} + ...,$$
$$y_1 = \frac{-x^4}{12} + \frac{x^5}{15} - \frac{13\,x^6}{360} + \frac{x^7}{63} - \frac{x^8}{224} + \frac{x^9}{2835} - \frac{37\,x^{10}}{907200} + ...,$$
$$y_2 = \frac{x^6}{360} - \frac{x^7}{315} + \frac{11\,x^8}{20160} + \frac{29\,x^9}{45360} - \frac{223\,x^{10}}{907200} + ...$$

The general solution given by series shape as follows:

$$y(x) = y_0(x) + y_1(x) + y_2(x) =$$
$$1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \frac{841\,x^8}{20160} - \frac{x^9}{15120} + \frac{209\,x^{10}}{25200} + ...$$

Table 1. Comparison of numerical between ADM $y = \sum_{i=0}^{2} y_i(x)$ and exact solution $y = e^{x^2}$

x	Exact	ADM	Absolute Error
0.0	1.0000	1.0000	0.0000
0.2	1.04081	1.04081	0.0000
0.4	1.171351	1.171351	0.0000
0.6	1.43333	1.43333	0.0000
0.8	1.89648	1.89637	0.00011

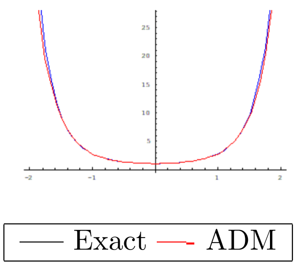

Fig. 1. The Approximation solution for ADM $y = \sum_{i=0}^{2} y_i(x)$ and Exact solution $y = e^{x^2}$

Problem 2. Put $m = 2, \ \ k = 2$, in equation (6), we have

$$y''' + 6y'' + 12y' + 8y = 27e^x + 8x^2 + 24x + 12 + \left(x^2 + e^x\right)^2 - y^2, \tag{23}$$

$$y(0) = y'(0) = 1, y''(0) = 3,$$

with the exact solutions $y(x) = x^2 + e^x$. Rewrite Eq.(23) as below:

$$Ly = 27e^x + 8x^2 + 24x + 12 + \left(x^2 + e^x\right)^2 - y^2, \tag{24}$$

where

$$L(.) = e^{-2x}\frac{d^3}{dx^3}e^{2x}(.)$$

and, the invers operator defin by

$$L^{-1}(.) = e^{-2x}\int_0^x\int_0^x\int_0^x e^{2x}(.)dxdxdxdx(.).$$

To take L^{-1} on Eq. (24) and using initial condition, we get:

$$y(x) = [1 + 3x + 4x^2]e^{-2x} + L^{-1}[27e^x + 8x^2 + 24x + 12 + \left(x^2 + e^x\right)^2 - y^2], \tag{25}$$

where

$$y_0 = [1 + 3x + 4x^2]e^{-2x} + L^{-1}[27e^x + 8x^2 + 24x + 12 + \left(x^2 + e^x\right)^2$$

$$y_{n+1} = -L^{-1}A_n, n \geq 0,$$

where A_n Adomian polynomials define by:

$$A_0 = y_0^2,$$

$$A_1 = 2y_0y_1.$$

The first few components are thus determined as follows:

$$y_0 = 1 + x + \frac{3\,x^2}{2} + \frac{x^3}{3} - \frac{x^4}{8} + \frac{7\,x^5}{40} - \frac{59\,x^6}{720} + \frac{31\,x^7}{720} - \frac{221\,x^8}{13440} + \frac{1997\,x^9}{362880} - \frac{263\,x^{10}}{172800} + ...,$$

$$y_1 = \frac{-x^3}{6} + \frac{x^4}{6} - \frac{x^5}{6} + \frac{29\,x^6}{360} - \frac{17\,x^7}{420} + \frac{139\,x^8}{10080} - \frac{x^9}{280} + \frac{451\,x^{10}}{1814400} + ...,$$

$$y_2 = \frac{x^6}{360} - \frac{x^7}{420} + \frac{3\,x^8}{1120} - \frac{169\,x^9}{90720} + \frac{2101\,x^{10}}{1814400},$$

$$y_3 =, \frac{-x^9}{15120} + \frac{x^{10}}{8640} + ...,$$

$$y(x) = 1 + x + \frac{3\,x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \frac{x^{10}}{3628800} + ...$$

We note that, through this example we get the exact solution.

Problem 3. To put in the Eq. (6) $m = 4, k = 1$ we get:

$$y^{(5)} + 5y^{(4)} + 10y^{(3)} + 10y^{(2)}) + 5y' + y = 32e^x + e^{4x} - y^4, \tag{26}$$

$$y(0) = y'(0) = y''(0) = y'''(0) = y''''(0) = 1,$$

Eq. (26) rewrite it as follows:

$$Ly = 32e^x + e^{4x} - y^4, \tag{27}$$

where L gives the formula:

$$L(.) = e^{-x}\frac{d^5}{dx^5}e^x(.)$$

and

$$L^{-1} = e^{-x} \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x e^x (.) dx dx dx dx dx.$$

To take L^{-1} on Eq. (27) and using initial condition, we get:

$$y(x) = [(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}) + (x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}) + (\frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{4}) + (\frac{x^3}{6} + \frac{x^4}{6}) + (\frac{x^4}{24})]e^{-x}$$
$$+ L^{-1}(32e^x + e^{4x}) - y^4,$$

where

$$y_0 = [(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}) + (x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}) + 4(\frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{4}) + (\frac{x^3}{6} + \frac{x^4}{6}) + (\frac{x^4}{24})]e^{-x}$$
$$+ L^{-1}(32e^x + e^{4x}),$$
$$y_{n+1} = -L^{-1}A_n, n \geq 0,$$

where A_n Adomian polynomials defind by

$$A_0 = y_0^4,$$
$$A_1 = 4y_0^3 y_1,$$

then

$$y_0 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{8} - \frac{x^5}{15} + \frac{x^6}{24} - \frac{29\,x^7}{2520} + \frac{5\,x^8}{1344} - \frac{29\,x^9}{72576} + \frac{719\,x^{10}}{3628800} + ...,$$
$$y_1 = \frac{-x^5}{120} + \frac{x^6}{720} - \frac{11\,x^7}{5040} - \frac{x^8}{4480} - \frac{19\,x^9}{60480} - \frac{19\,x^{10}}{201600} + ...,$$
$$y_2 = \frac{x^{10}}{907200} + ...,$$

The general solution given by series shape as follows:

$$y(x) = y_0(x) + y_1(x) + y_2(x) =$$
$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{8} - \frac{3\,x^5}{40} + \frac{31\,x^6}{720} - \frac{23\,x^7}{1680} + \frac{47\,x^8}{13440} - \frac{37\,x^9}{51840} + \frac{127\,x^{10}}{1209600} + ...$$

Table 2. Comparison of numerical between ADM $y = \sum_{i=0}^{2} y_i(x)$ and exact solution $y = e^x$

x	Exact	ADM	Absolute Error
0.0	1.0000	1.0000	0.0000
0.1	1.10517	1.10518	0.00001
0.2	1.2214	1.22151	0.00011
0.3	1.34986	1.35036	0.0005
0.4	1.49182	1.49325	0.00143
0.5	1.64872	1.65188	0.00316
0.6	1.82212	1.82805	0.00593
0.7	2.01375	2.02369	0.00994
0.8	2.22554	2.24088	0.01534
0.9	2.4596	2.48182	0.02222

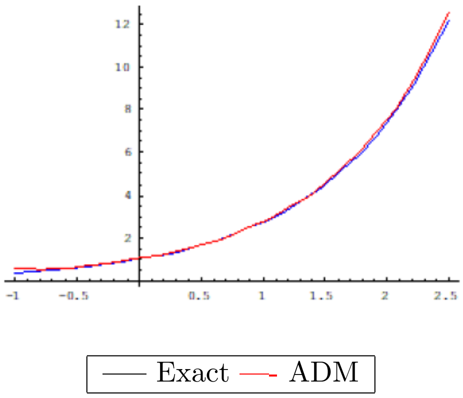

Fig. 2. The Approximation solution for ADM $y = \sum_{i=0}^{2} y_i(x)$ and Exact solution $y = e^x$.

Problem 4. Consider the linear problem when $m = 6, k = \frac{1}{2}$ in Eq.(6) we get

$$y^{(7)} + 7(\frac{1}{2})y^{(6)} + 21(\frac{1}{2})^2 y^{(5)} + 35(\frac{1}{2})^3 y^{(4)} + 35(\frac{1}{2})^4 y^{(3)} + 21(\frac{1}{2})^5 y^{(2)} + 7(\frac{1}{2})^6 y' + (\frac{1}{2})^7 y =$$

$$5040 + 17640\,x + 13230\,x^2 + 3675\,x^3 + \frac{3675\,x^4}{8} + \frac{441\,x^5}{16} + \frac{49\,x^6}{64} + \frac{x^7}{128}, \tag{28}$$

$$y(0) = y'(0) = y''(0) = y'''(0) = y''''(0) = y'''''(0) = y'''''' = 0$$

Eq.(28) can be write as follows:

$$Ly = 5040 + 17640\,x + 13230\,x^2 + 3675\,x^3 + \frac{3675\,x^4}{8} + \frac{441\,x^5}{16} + \frac{49\,x^6}{64} + \frac{x^7}{128}. \tag{29}$$

The differential operator L for Eq.(28) is:

$$L(.) = e^{\frac{-1}{2}x} \frac{d^7}{dx^7} e^{\frac{1}{2}x}(.),$$

and

$$L^{-1}(.) = e^{-\frac{1}{2}x} \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x \int_0^x e^{\frac{1}{2}x}(.)dxdxdxdxdxdxdx.$$

Applying the L^{-1} on Eq.(29) and using initial conditions, we get the exact solution .

$$y(x) = x^7.$$

4 Conclusion

In this paper, we presented a new modification of the ADM and tested its reliability. Through the illustrative examples presented in this paper, we found that by using the new differential operator, we get solutions that approximate the exact solution. We can conclude that the modified ADM is a reliable and effective method to solve initial value problems of higher order.

Competing Interests

Authors have declared that no competing interests exist.

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