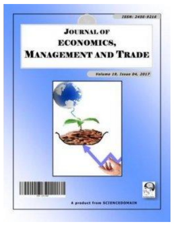

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The Mechanism of the Links between Growth and Volatility

Kcodgoh L. Edgeweblime[1*]

[1]*Department of Economics, American Institute of Africa, Lomé, Togo.*

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This paper has investigated the signs of the relationship between growth and volatility in a new environment set by changes in assumptions of the Ramsey model to allow technological progress to become endogenous.
A generation's utility maximization is obtained through externalities trade named "multidimensional trade". In contrast to the effects on long-run growth in the AK model where an improvement in the level of technology, A, which raises the marginal and average products of capital, also raises the growth rate and alters the saving rate, we found a greater willingness to hoard down or an improvement in the level of technology shows up in the long-run as higher levels of capital (unnatural resources) and output per effective worker but in no change in per capita growth rate. The steady state results of the working of diminishing returns to inputs in technology production function. This has leaded to a reformulation of Heckscher-Ohlin trade model: Productive factors that exist in abundance in a generation and that are not intensively used to produce goods and services in that generation are exported to other generations in exchange for scarce productive factors intensively used to produce goods and services that should be scarce in the generation. The goods and services with weak consumption are indirectly exported from one generation to others, whereas goods and services with high consumption are indirectly imported from other generations. Therefore

**Corresponding author: E-mail: ldslome1@yahoo.fr;*

Ramsey model becomes a particular case of multidimensional trade (when externalities are internalized). In that case, the tendency for saving rates to rise or fall with economic development affects the traditional dynamics, that is why, in our framework, intergenerational and international leveling out of all prices (goods and productive factors include techniques) should restore neoclassical assumptions: this is the main mechanism of the links between growth an volatility.

Keywords: Production possibilities frontiers; growth volatility; international trade elasticity; intergenerational trade; intergenerational prices leveling out.

1. INTRODUCTION

In the Ramsey model, the tendency for saving rates to rise or fall with economic development affects the traditional dynamics with “zero cost” of technological progress is still controversial. I attempt in this paper to modify the Ramsey model in two respects: First, I allow technological progress to become endogenous in order to exclude dynastic altruism in an intergenerational trade based on competitive markets and twice, this intergenerational trade should interact with international trade viewed as multiple current generations exchanging goods with each other.

Starting from Ramsey growth model, I will study, the sign of the relationships between growth and volatility. In fact, if a generation decides to use more of natural resources(negative externalities) today, Pareto-optimality condition requires to compensate that overconsumption by an equivalent value of positive externalities (unnatural resources) on future generations. Caselli and Coleman (2000) define a country’s technology as a combination of unskilled and skilled labor and capital efficiencies. They found a negative cross-country correlation between the efficiency of unskilled labor and the efficiencies of skilled labor and capital. In addition, they interpret this link as proof of the existence of a World Technology Frontier in which increases in the efficiency of unskilled labor are obtained at the cost of efficiency declines in skilled labor and capital. Therefore, intergenerational technology frontier should play the same function for intergenerational trade to restore Pareto-optimality. This kind of trade should be focused on natural resources against unnatural resources (techniques, institutions, durable infrastructures and capital). If an intergenerational leveling-out of the prices of goods and factors is not realized, changes in the supply of goods and factors become unbalanced, inducing movements in generations’ and nations’ production possibility frontier (which can be clipped as) (PPF), thus causing fluctuations.

The purpose of this paper is to investigate the role of non Pareto-optimal Walrasian equilibria in the exchange of externalities between countries and/or between generations. This brings into focus the following questions: Does an algebraic sum of multidimensional trade scale effects impact the relationships between world PPF’s and intergenerational PPF’s? In other words, can disequilibria in the exchange of externalities between countries and generations explain the relationship between growth and volatility? In this paper we expand on Bajona and Kehoe’s [1] theoretical model building a foundation and integrating and testing multidimensional trade.

All resources (natural and unnatural) allocated through suboptimal and ‘optimal’ choices (trade relationships) are crucial to the relationships between growth and volatility. A country can exchange goods and services with other countries, while each generation can also exchange resources with adjacent generations. This latter exchange can be optimal or suboptimal. The image of international interdependencies is established, and as in the situation where nothing is created and nothing is lost, each generation (or country) generates effects (or shocks) on other generations (or countries). This is done permanently; so each generation’s (or country’s) PPF is continually moving around the fixed world frontier. These movements impact on generational and country trade through gained or lost comparative advantages. As international trade intensity reduces with distance, exchanges between generations decline with both time and distance. Our research presents three models; an international trade model in an environment of unrelated generations, an intergenerational trade model in an environment of autarkic conditions and a multidimensional trade model which is a combination of the international and intergenerational trade models.

This paper is presented with section one providing background and motivations. The second section addresses our model’s background and motivation and section three examines the model setup, tests and solutions.

Finally, section four presents the paper's conclusions.

2. THE BACKGROUND

In the traditional economic theory, growth is supposed to play no role in economic volatility, however, three papers presented in the early 1980's changed the understanding of that important issue. Nelson and Plosser [2] find that the movements in the GNP tend to be permanent. Kydland and Prescott [3] uncover skills for analyzing economic volatility and integrating growth and volatility (fluctuations). Lucas [4] shows that the possible returns from understanding business-cycles are trivial compared to these from understanding growth assuming that growth and business cycle volatility are unrelated (the standard dichotomy in macroeconomics).

According to Ragchaasuren (2006), the models that follow Shumpeter (1942), where the mechanism is based on "creative destruction" show a positive relationship between growth and volatility. For example, in Aghion and Saint-Paul [5], productivity change is assumed to be the result of purposeful (internal) learning through deliberate actions which substitute for production activities. Under such circumstances, the resources allocated to productivity improving activities are a convex function of the state of the economy and hence the average productivity increases as volatility increases. On the other hand, the models that follow Arrow [6], where the mechanism of technological change takes the form "learning by-doing" show that the relationship between growth and volatility tends often(but not always) to be negative. For example, in Martin and Rogers [7,8], productivity change takes place through serendipitous (external) learning through non-deliberate actions which are complements to production activity. In this case, the factor through which expertise, knowledge and skills are acquired and disseminated is a concave function of the shocks, so that increased volatility decreases growth. By incorporating the above two conflicting mechanisms for endogenous technological change, Blackburn and Galindev [9] shows that the any shocks can have a permanent effect on output if it changes the amount on which productivity improvements depend. For Aghion and Howith (1998), Dinopoulos and Thompson (1998), Jones (1995), Kortum (1997), Peretto (1998), Segerstrom (1998) and Young 1998 there exists a positive linkage between productivity growth rate and the share of R&D in GDP. For example Black [10] argues that countries may have a choice between high-variance, high expected returns technologies because countries with high average growth would also have high variance. Conversely, Bernanke [11], Pindyck (1991), Aizenman and Marion [12], Ramey and Ramey [13,14] argued that there is a negative association between productivity growth rate and the share of R&D in output. If lower current output affects resources' accumulation, then growth is adversely affected. For example, the theoretical analysis suggests that, if there is an irreversibility in investment, then an increasing volatility can lead to lower investment Bernanke [11], Pindyck (1991), etc.). See Ramey (2012) who investigated that increases in government spending stimulate private activity. She found that in most cases private spending falls significantly in response to an increase in government spending. See also Bean [15], Fatas [16], King et al. [17], Jones et al. [18] for permanent effects of temporary real shocks, and Stadler [19], Pelloni [20], Blackburn [21] and Blackburn and Pelloni [22] for permanent effects of temporary nominal shocks See also Caballero and Hammour [23] for a related contribution on this subject (see Blackburn [21] for a contrasting result in this approach). Relationship between growth and volatility is more likely to be positive (negative) if technological change is predominantly driven by internal (external) learning. In contrast to the above, some models in which knowledge is created under the assumption of learning-by-doing suggest alternative relationships between growth and volatility. According to De Hek [24] and Smith [25], the relationship between long-term growth and short-term cyclical volatility depends on the household's attitude towards risk as measured by the curvature of the utility function. Specifically, the more (less) risk-averse is an agent, the more likely it is that increased uncertainty will have a positive (negative) effect on long-run growth. Jones et al. [18] considers the same issue in a different framework in which growth is the result of constant returns to reproducible factors – physical and human capital – that are purely rival (and not due to the accumulation of non-rival knowledge via learning-by-doing) and reaches the result the same as above. Blackburn and Pelloni [22] investigates the correlation between the growth and volatility depends on the nature of the shocks under the assumption of an imperfect labor market. Long-run growth is positively correlated with the volatility of the real shocks and negatively correlated with the volatility of the nominal shocks.

All the resources (natural and non-natural) allocation through suboptimal and "optimal" choices (trade relationships) is the key responsible of the nature of the relationship between growth and volatility. As each country can exchange goods and services with other countries, each generation exchanges also resources (natural and non-natural) with neighbor generations. This latter exchange can be optimal or suboptimal at the image of international interdependencies and - as in the nature nothing is created and nothing is lost- each generation (country) generates effects (shocks) on other generations (countries) in a permanent way so that each generation or country production possibilities frontier is continually moving around the whole world frontier which is fix. These movements impact on generations and countries trade through comparative advantages gained or lost. As trade intensity is internationally reducing with the distance, each generation exchange with other generations reduces with timely distance.

3. THE MODEL

3.1 Growth Models with Consumer Optimization (The Ramsey Model)

A more complete picture of growth model needs to allow for the path of consumption and the saving to be determined by optimizing households and firms that interact on competitive markets. The reasoning is based on the infinitely lived households that choose consumption and saving to maximize their dynastic utility, subject to an intertemporal budget constraint, a key element in Ramsey model [26], refined by Cass [27] and Koopmans [28].

In this model, the saving rate is no longer constant but is determined by the per capita capital stock, k. Therefore, the average level of saving rate is pined down so that the saving rate can rise or fall as the economy develops. The saving rate is also determined by interest rate, tax rates and subsidies. Ramsey model still have convergence property under fairly general conditions, so that the Solow-Swan model with a constant saving rate is here a special case.

3.1.1 Households

The family size at time t is L(t) = e^{nt} (1)

If C(t) is the total consumption at time t, then c(t)≡ C(t)/L(t) is consumption per adult person

Each household wishes to maximize overall utility, U, as given by

$$U = \int_0^{\infty} u[c(t)]e^{nt}\ e^{\rho t}dt \qquad (2)$$

This formulation assumes that the household utility at time 0 is a weighted sum of all future flows of utility u©

Since each person works one unit of labor services per unit of time, the wage income per adult person equals w(t). The total income received by the aggregate of household is therefore, the sum of labor income, w(t). L(t), and asset income , r(t). (Assets). Households use the income they do not consume to accumulate more assets:

$$\frac{d(Assets)}{dt} = \text{r. (Assets)} + \text{wL} - \text{C} \qquad (3)$$

Which can be transformed as: $\dot{a}$ =w +ra-c-na (4)

If each household can borrow an unlimited amount at the going interest rate, r(t), it has the incentive to pursue a form of chain letter or Ponzi game. The household can borrow to finance current consumption and then use future borrowings to roll over the principal and pay all the interests. In this case, the household debt grows forever at the rate of interest, r(t).

To rule out chain-letter possibilities, we assume that the credit market imposes a constraint on the amount of borrowing. The appropriate restriction turns out to be that the present value of assets must be asymptotically nonnegative, that is,

$$\lim_{t\to\infty}\{a(t).exp[-\int_0^t[r(v) - n]dv]\} \geq 0 \qquad (5)$$

This constraint mean that, in the long run, a household's debt per person cannot grow as fast as r(t)-n

The household's optimization problem is to maximize U in equation (2), subject to the budget constraint in equation (4).

The first order conditions

$$\frac{\partial J}{\partial c} = 0 \Rightarrow v = u'(c)\ e^{-(\rho-n)t}$$

$$\dot{v} = \frac{\partial J}{\partial a} \Rightarrow \dot{v} = -(r-n).v$$

We therefore follow the common practice of assuming the functional form

$$\mathrm{u(c)} = \frac{c^{1-\theta}-1}{1-\theta} \qquad (6)$$

with the first order conditions we have: U = $\int_0^\infty e^{-(\rho-n)t}.[\frac{c^{(1-\theta)}-1}{1-\theta}]\mathrm{dt}$

Where $\theta > 0$, so that the elasticity of marginal utility equals the constant $-\theta$. The elasticity of substitution of this utility function is the constant δ=1/θ, hence this form is called the constant intertemporal elasticity of substitution (CIES) utility function.

The form of u(c) in equation (6) implies that the optimality condition from equation (5) simplify to

$$\dot{c}/c \;\; (1/\theta).(\mathrm{r}-\rho) \qquad (7)$$

We see that the relation between r and ρ determines whether households choose a pattern of per capita consumption that rises over time, stays constant, or fall over time. A lower willingness to substitute intertemporally implies a smaller responsiveness of $\dot{c}/c$ to the gap between r and ρ.

Transversality condition

The consumption function

$$\left[\check{r}(t) = \left(\frac{1}{t}\right).\int_0^t r(v)dv\right]$$

$$\mathrm{a(T)} \;\; e^{-[\check{r}(T)-n]T} + \int_0^T \mathrm{c(t)}.e^{-[\check{r}(t)-n]t}\,dt = \mathrm{a(0)} + \int_0^T w(t)\,e^{-[\check{r}(t)-n]t}dt$$

$$\mathrm{a(T)} \;\; e^{-[\check{r}(T)-n]T} + \int_0^\infty \mathrm{c(t)}.e^{1/\theta/[\check{r}(t)-\rho]t}\,dt = \mathrm{a(0)} + \int_0^\infty w(t)\,e^{-[\check{r}(t)-n]t}dt = a(0) + \widetilde{w(0)} \qquad (8)$$

The consumption function is given by

$$\mathrm{C(t)} = \mathrm{c(0)}.e^{1/\theta/[\check{r}(t)-\rho]t} \qquad (9)$$

The substitution of this result for c(t) into the intertemporal budget constraint in equation (8) leads to the consumption function at time 0:

$$\mathrm{c\,(0)} = \mu(0).\,[\mathrm{a(0)} + \bar{w}\acute{(0)}]$$

Where μ(0), the propensity to consume out of wealth, is determined from

$$[1/\,\mu(0).] = \int_0^\infty e^{\check{r}(t)\left(1-\theta\right)/\theta-\frac{\rho}{\theta}+n\big]t}\,dt \qquad (10)$$

An increase in average interest rates, $\check{r}(t)$, for a given wealth has two effects on the marginal propensity to consume in equation (10). First higher interest rate increases the cost of current consumption relative to future consumption, an intertemporal substitution effect that motivates households to shift consumption from the present to future. Second higher interest rates have an income effect that tends to raise consumption at all dates. The net effect of an increase in $\check{r}(t)$ *on* μ(0) depends on which of the two forces dominates.

3.1.2 Firms

The production function is:

$$\mathrm{Y(t) = F[K(t), L(t),T(t)]} \qquad (11)$$

K(t), the capital Input, L(t), labor input and T(t), the level of technology which is assumed to grow at a constant rate x>0.

F(.) satisfies the neoclassical properties.

If $\hat{L}$ =L.T(t), we have:*

$$\mathrm{Y = F(K,} \hat{L}) \qquad (12)$$

$$\text{If } \hat{y} = \mathrm{Y}/\hat{L} \text{ and } \hat{k} = \mathrm{K}/\hat{L} \qquad (13)$$

The production function becomes

$$\hat{y} = \mathrm{f}(\hat{k}) \qquad (14)$$

It is demonstrated that each firm who takes r and w as given maximizes profit for given $\hat{L}$

By setting f'($\hat{k}$) = r +δ

$$\text{At the equilibrium } \dot{\hat{k}} = \mathrm{f}(\hat{k}) - c - (x + n + \delta).\,\hat{k} \qquad (15)$$

The transversality condition can be written:

$$\lim_{t\to\infty} \;\; \{\hat{k}.\exp\,(\int_0^t [f'(\hat{k}) - \delta - \mathrm{x} - \mathrm{n}]\mathrm{dv}\} \qquad (16)$$

3.1.3 Ramsey model of consumer optimization versus solow-swan neoclassical model

3.1.3.1 The foundations

Ramsey model considers that technology grows at a constant rate so that we have posed $\hat{L}$ =L.T(t) to transform production function into Y =F(K, $\hat{L}$) assuming that technological progress is labor augmenting. This statement leads to various problems in Solow-Swan model:

First, it is demonstrated that in many situations, technological progress changes marginal products so that constant return to scale cannot be stated. In an optimizing model where each firm operates on a competitive market, a technological progress generally leads to substitute the input that the price becomes low (techniques effect) to the input that the price increases or stays constant, in order to maximize its profit.

Twice, there is no reason for technological progress to be only labor augmenting. If, as the model states, the saving rate is not exogenous, firms in an optimizing world will invest in the kind of R&D which is supposed to solve a problem. Ragchaasuren (2006), has demonstrated, the models following Schumpeter [29], where the mechanism is based on creative destruction, the factors through which expertise, knowledge and skills are acquired and disseminated, is a concave function of the shocks; By incorporating the two conflicting mechanisms for endogenous technological change, Blackburn and Galindev [9] show that any shocks can have a permanent effect on output if they change the amount on which productivity improvements depend.

3.2 A Model of Infinitely Lived Consumers and Overlapping Generations

In their model Bajona and Kehoe [1] consider n countries which differ in their population size and their initial endowments of capital. Each country can produce three goods: two traded goods- a capital intensive good and a labor-intensive good- and a non-traded investment good. The technologies available to produce these goods are the same across countries. Each traded good j, j = 1, 2, is produced by using capital and labor according to the production function

$$Y_j = \Phi_j(k, l) \tag{17}$$

The function is increasing, concave, continuously differentiable and homogenous of degree one.

Producers minimize costs taking prices as given and earn zero profits.

Good 1 is relatively capital intensive and there is no capital intensity reversal and the investment good is produced using the two traded goods: $x = f(x_1, x_2)$

Capital depreciate at the rate δ, $1 \geq \delta > 0$

The first order conditions for profit maximization are:

$$P_1 \geq qf_1(x_1, x_2), = \text{ if } x_1 > 0 \tag{18}$$

$$P_2 \geq qf_2(x_1, x_2), = \text{ if } x_2 > 0 \tag{19}$$

Where q is the price of investment good

Labor and capital are not mobile across countries, but are mobile across sectors within a country.

3.2.1 Infinitely lived consumers

The environment is characterized with infinitely lived consumer-workers, each country i, i = 1, …, n, has a continuum of measure L^i of consumers, each of whom is endowed with $k_0^i > 0$ units of capital in period 0 and one unit of labor at every period, which is supplied inelastically. Consumers have the same utility functions, within countries and across countries. In each period, the representative consumer in country i decides how much to consume of each of the two traded goods in the economy, c^i_{1t}, c^i_{2t}, how much capital to accumulate for the next period, k^i_{t+1}, and how much to lend b^i_{t+1}. Consumers derive their income from wages, w_{it}, returns to capital, r_{it}, and return to lending, rb_{it}. The representative consumer in country i solves the problem

$$\max \sum_{t=0}^{n} \binom{n}{k} \beta^t u(ci1t, ci2t) \tag{21}$$

s.t. $p_1 c_{i1} + p^2{}_t c_i{}^2{}_t + q_{i1} x_{it} + b_{it}+1 \leq w_{it} + r_{it} k_{it} + (1+rb_{it}) b_{it}$

$k_{it}+1 - (1-\delta) k_{it} \leq x_{it}$

$c_{ijt} \geq 0$, $x_{it} \geq 0$, $b_{it} \geq -B$

$k_i 0 \leq k_{-i0}$, $b_{i0} \leq 0$

The period utility function $u(c_1, c_2)$ is homothetic, strictly increasing, strictly concave, and continuously differentiable.

The first order conditions of this consumer problem (21) imply that

$$\frac{u2(ci1t, ci2t)}{u1(ci1t, ci2t)} = \frac{p2t}{p1t} \tag{22}$$

Endowment of labor per worker differs across countries, as long as these differences remain constant over time.

The feasibility conditions for factor and for investment good are

$k_{i1t}+k_{i2t} \le k_{it}$ (23)

$l_{i1t}+l_{i2t} \le l$ (24)

$k_{it+1} -(1-\delta)k_{it} \le x_{it}$ (25)

3.2.2 Overlapping generations

A new generation of consumer-workers is born in each period in each country. Consumers in generation t, t= 0,1, …are born in period t and live for m periods. Each of these generations in country i has a continuum of measure L^i of consumers. Each consumer is endowed with $\bar{l}^h$ units of labor supplied inelastically. Consumers can save through accumulation of capital and bonds. Consumers are born without any initial endowment of capital and bonds. The representative consumer born in country i in period t, t= 0,1,…solves

$$\max \sum_{h=1}^{m} ßh \;\; \mathrm{uh(cit1t + h - 1cit2t + h - 1)} \qquad (26)$$

s.t. $p_{1t+h-1}c^{it}_{1t+h-1} + p_{2t+h-1}c^{it}_{2t+h-1} + q^{it}_{+h-1}x^{it}_{t+h-1} + b^{it}_{t+h}+1 \le w^{i}_{t+h-1}\bar{l}^{h} + r^{i}_{t+h-1}\;\; k_{it}+(1+r^{bi}_{t+h-1})b^{it}_{t+h-1}+r^{i}_{t+h-1}k^{it}_{t+h-1}$

$k^{it}_{t+h}-(1-\partial)k^{it}_{t+h-1} \le x^{it}_{t+h-1}$

$c^{it}_{j+h-1} \ge 0,\ x^{it}_{ht+h-1} \ge 0$

$k_{it}0 \le k^{-it}_{0,}\ b^{it}_{t} \le 0,\ x^{it}_{t+m-1} \ge -(1-\partial)k^{it}_{t+m-1}, b^{it}_{t+m} \ge 0$

u_h is utility function in period of life h.

For every h, h= 1,…,m, the utility function uh(c1,c2) is homothetic, strictly increasing, strictly concave, and continuously differentiable, with $\lim_{cj\to 0} uhj(c_1,c_2)= \infty$ $\lim_{cj\to\infty} uhj(c_1,c_2)=0$

There are m-1 generations of initial old consumers alive in period 0. Each generation s, s= -m+1,…,-1, in country i has a continuum of measure L_i of consumers, each of whom lives for m+s periods and is endowed with $\bar{l}^{h-s}$ units of labor in period h, h=1,…,m+s.

The representative consumer of generation t, t=-m+1,…,-1, in country I solves

$$\max \sum_{h=1}^{m} ßh \;\; \mathrm{uh(cit1t + h - 1cit2t + h - 1)} \qquad (27)$$

s.t. $p_{1t+h-1}c^{it}_{1t+h-1} + p_{2t+h-1}c^{it}_{2t+h-1} + q^{it}_{+h-1}x^{it}_{t+h-1} + b^{it}_{t+h} \le w^{i}_{t+h-1}\bar{l}^{h} + (1+r^{bi}_{t+h-1})b^{it}_{t+h-1}+r^{i}_{t+h-1}k^{it}_{t+h-1}$

$k^{it}_{t+h}-(1-\partial)k^{it}_{t+h-1} \le x^{it}_{t+h-1}$

$c^{it}_{j+h-1} \ge 0,\ x^{it}_{ht+h-1} \ge 0$

$k^{it}_{0} \le k^{-it}_{0,}\ b^{it}_{t} \le 0,\ x^{it}_{t+m-1} \ge -(1-\partial)k^{it}_{t+m-1}, b^{it}_{t+m} \ge 0$

3.2.3 Equilibrium

There are n countries of different size, L_i, i=1,…,n and different initial endowments of capital and bonds: k^j_0 and b^j_0, i=1,….,n in the environment with infinitely lived consumers and k^{is}_0 and b^{is}_0, s=-m+1,…,-1,i=1,…,n in the environment with overlapping generations. An equilibrium is sequences of consumptions, investments, capital stocks, and bonds holdings$\{c_{i1t},c^{i2t},x^i_t,k^i_t,b^i_t\}$ in the environment with infinitely lived consumers and $\{c^{is}_{1t},c^{is}_{2t},x^{is}_t,k^{is}_t,b^{is}_t\}$,s=t-m+1,…t, in the environment with overlapping generations, output and input for each traded industry, $\{y^i_j,k^i_j,l^i_j\}$, j=1,2, output and inputs for the investment sector $\{x^i_t,x^i_t,x^i_{2t}\}$, and prices $\{p_{1t},p_{2t},q^i_t,w^i_t,r^i_t,r^{bi}_t\}$, i=1,…n, t=0,1,2,…, such that

Given prices $\{p_{1t},p_{2t},q^i_t,w^i_t,r^i_t,r^b_{it}\}$, the consumption and accumulation plan$\{c^i_{1t},c^i_{2t},x^i_t,k^i_t,b^i_t\}$ solves the consumers problems (4) in the environment with infinitely lived consumers and the consumption and accumulation plan $\{c^{is}_{1t},c^{is}_{2t},x^{is}_t,k^{is}_t,b^{is}_t\}$ solves the consumers' problems (21) and (22) in the environment with overlapping generations.

Given prices $\{p_{1t},p_{2t},q^i_t,w^i_t,r^i_t,r^{bi}_t\}$, the production plan $\{y^i_j,k^i_j,l^i_j\}$ and $\{x^i_t,x^i_t,x^i_{2t}\}$satisfy the cost minimization and zero profit conditions.

The consumption, capital stock, $\{c^{is}_{1t},c^{is}_{2t},x^{is}_t,k^{is}_t,b^{is}_t\}$or $\{c^{is}_{1t},c^{is}_{2t},x^{is}_t,k^{is}_t,b^i_{st}\}$, and production plans, $\{y^i_j,k^i_j,l^i_j\}$and $\{x^i_t,x_{it},x^i_{2t}\}$,satisfy the feasibility conditions in infinitely lived consumers and overlapping generations environment.

A steady state is consumption levels, an investment level, a capital stock, and bond holding, $(\hat{c}^i_1,\hat{c}^i_2\ x^i,\ k^i,b^i)$ in the environment with infinitely lived consumers and , $(\hat{c}^{is}_1,\hat{c}^{is}_2\ x^{is}, k^{is},b^{is})$, s=1,….m, in the environment with overlapping, output and inputs for each traded industry $\{y_{ij},k^i_j,l^i_j\}$, j=1,2, output and inputs for the investment sector, $\{x^i_t,x^i_t,x^i_{2t}\}$ and prices $\{p_{1t},p_{2t},q^i_t,w_{it},r^i_t,r^b_{it}\}$, i= 1,…n, that satisfy the conditions of competitive equilibrium for appropriate initial endowments of capital and bonds in the environment of infinitely lived consumers and overlapping generations. The Bajona and Kehoe typical model (2006) that is in concern here ends in equation (27).

3.2.4 Steady states

In a model of infinitely lived consumers that satisfies essential conditions have price equalization in any nontrivial steady state. In that model, we have a continuum of steady states. There is international trade in every steady state. As the world converges to its steady state, each country converges to a steady state that depends on its initial endowments of capital relative to the world average.

3.2.5 Infinitely lived consumers and overlapping generations model's problems

The absence of technological progress in the model implies that intergenerational trade has many problems: 1) The constant returns production function at the aggregate level can reflect learning-by-doing and spillovers of technology but is not Pareto optimal; 2) There is no attempt to internalize- within generations and countries- spillovers of technology; 3) Convergence to steady states and prices equalization indicate that countries and generations are strictly identical and, therefore intergenerational and international trade is impossible; 4) The picture of properties of dynamic Heckscher-Ohlin models poses the problems of dynamic inefficiency. Fundamentally, we should admit that the first generations have external effects (positive or negative) on the following generations. These effects are: technological progress obtained by learning by doing or in the firms of R&D, knowledge produced by universities, institutions, durable infrastructures and physical capital … A reasonable intergenerational trade should be based on negative external effects (overconsumption of natural resources, bad institutions, bad knowledge …) against positive external effects. The sustainable development principle is that current generations should satisfy their needs without diminishing the capacities of the future generations to satisfy their own needs. The most important measurable and positive external effect of current generation on future generations is technical knowledge (techniques, institutions … produced by universities and firms of R&D. Hence, technological progress causes reversibility in capital or labor intensity in the process of production.

This model ignores intergenerational and international trade interferences. Intergenerational trade is one of the main reasons why some countries are developed and others not. The hypothesis of consumer-workers fixed endowments cannot be stated. Several other hypothesis of this model should be reviewed. (Revisited)

3.3 Setup of the Model

3.3.1 Behavior of households and firms

3.3.1.1 The international trade

In this part of the model because the generations are unrelated the overlapping generations' hypothesis does not apply (the intergenerational autarky condition). Each country has initial different endowments (at the beginning of the country's life) composed of natural and unnatural resources. Natural resources (the physical environment) and unnatural resources (other resources) are the productive factors in the economy. Each country has its own comparative advantages.

China is well endowed in natural resources and the United States has unnatural resources. At the start of international trade China will export wheat (indirect, natural resources). China is producing natural resource intensive goods. China will import DVDs (indirect, unnatural resources) from the United States, which is producing relatively intensive unnatural resources.

These conditions and concavity imply

$n_w / N_w > n_d / N_d$.

Through these conditions, we can establish the following analysis based on common neoclassical understandings.

The neoclassical Heckscher–Ohlin model (H–O model) (1933) states: "that countries export goods that require in their production the intensive use of productive factors found in abundance locally and goods where production demands the inverse proportions of the same factors are imported."The free trade production level is W. Consumption and the world equilibrium is noted at X. At point X perfect equilibrium of production and consumption for the two countries is realized (achieved). Each country improves its utility when passing from the lower indifference curve to the upper curve (one) At this point, the quantities of produced and consumed goods for both countries are determined.

Consider a world containing two countries (China and the United States), where each country has only two generations (US current generation G_c and US future generation G_f, China current generation G^*_c and China future generation G^*_f), two goods (wheat and DVDs), and two productive factors (natural resources and unnatural or produced resources). Wheat is natural resources intensive and DVDs are unnatural resources intensive. Countries and generations have differing natural and unnatural resources. Natural resources include the physical environment and can be converted to an equivalent measure of surface area per capita. Unnatural resources can also be converted to a uniform measure. This is a long run physical capital per capita (knowledge, techniques, physical capital, institutional capital, and traditions). Natural resources are not variable over time while unnatural resources continually increase at a rate ∂. Final goods are mobile through countries but not through generations, whereas the productive factors are mobile through generations but not through countries. The mobility of the productive factors is obtained through the exchange of positive externalities against negative externalities. Positive externalities are produced when unnatural resources survive into another generation. Negative externalities are created when a generation over-consumes a natural resource. Bajona and Kehoe's hypothesis compatibles are accepted along with what is described above.

These conditions and concavity imply

$n_w/N_w > n_d/N_d$.

Each international movement induces a consecutive wave of income flow across the countries.

The initial endowment ratio of country i (with y_i = GDP) is equal to y_i/Y= ý. Y is world income.

Country i should use its y_i/Y of natural and unnatural resources to produce and decide which goods to consume and which to export (saving) in exchange for imports (investment). These exports and imports will follow many industrial processes (convergent, divergent, complex, mono-industrial and multi-industrial processes) and affect global economic growth. World income distribution flows from Y to Y'. National income becomes y'_i and y'_i/Y'= ý' becomes the new wealth endowment ratio.

Each country uses its new resources to produce goods and services for their own consumption and to export. At the end of the first process, countries will have in co-ownership

$$\Delta Y - \Delta Y[\beta + \delta(1-\beta)]. \quad (28)$$

β is the internal absorption ratio (absorption by income unit) while δ is the economy's openness ratio ($\beta = \frac{Ci+Ii+Gi}{yi}$, $\delta =\ .\frac{xi+mi}{yi}$).

C_i is national consumption, I_i is national investment, and G_i is national public consumption.

At the beginning of the second wave, the additional income remains $\Delta Y[(1-\beta)(1-\delta)]$ (29)

The second wave of processes generates unnatural resources. Wealth generation is calculated as

$$\Delta Y[(1-\beta)(1-\delta)][(1-\beta)(1-\delta)] = \Delta Y[(1-\beta)(1-\delta)]^2 \quad (30)$$

At the end of the wave of processes, the impact on the global income equals the sum of geometric progression with a gain less than one. This sum can be given as the following expression:

$$\frac{\Sigma\Delta yit}{Y[(1-\beta)(1-\delta)]} = \frac{\Sigma\Delta yit}{[\beta+\delta(1-\beta)]} = \Sigma\Delta Yit \quad (31)$$

The optimal growth multiplier is $\frac{1}{[\beta+\delta(1-\beta)]}$

At each point in time, consumers in country i decide how much of each of the two goods to consume, the quantity of unnatural resources to accumulate for the next generation and, consequently, the quantity of natural resources to borrow from coming generations.

Each wave of exchange generates income fluxes through countries, which follow sinusoidal functions, represented as:

$$\Sigma\Delta yit = yi0\cos(\mathrm{Wijt} - (\varphi 1 + yi1\cos(\mathrm{Xit} - \varphi 1). \quad (32)$$

$$\Delta Yt = \Delta Yit = \Sigma\Sigma\ \Delta yit \quad (33)$$

Periodic function study indicates each periodic movement with P, as the period is a sum of sinusoidal movements and with p, $\frac{p}{2}, \frac{p}{3}$, as the period. These represent the harmonics of the system.

Following (basing on) Grossman and Helpman's (1991b) proposition, $w_{ij}(t)$ is modeled as the ratio of country i's total trade with country j. This ratio is calculated by country i's bilateral exports and imports divided by country i's output aggregate. This is represented:

$$wij = \frac{\frac{Pj(t)}{Pi(t)} \quad Li(t)gij(t)+Lj(t)gji(t)}{Li(t)yi(t)} i \neq j \quad (34)$$

g_{ig} (t) represents country i's real per capita consumption of country j's factors. $P_i(t)$ is the price of factor i while $L_i(t)$ is country i's population, at each time period, t.

We now define a_{ij} (where $0 \le a_{ij} \le 1$) as a constant, representing country j's share of accessible natural resources which can be consumed by country i as part of its own unnatural resources. Using (with regard to) Abramovitz's social capability (1986), a_{ij} determines a country's potential to adopt existing technologies. Using (basing on) these definitions, the accumulation of unnatural resources in country i may be written as

$$X^*_i(t) = \Phi[\Sigma a_{ij} w_{ij}(t) X_j(t)\ ] + (\Phi - \delta_X) X_i(t)\ . \quad (35)$$

Where Φ represents the common productivity parameter and δ_X is the rate of depreciation of unnatural resource stock (either obsolete or otherwise). It is assumed that $\Phi \ge \delta_X > 0$.

The measure of country C_i's exchange with country C_j, w_{ij} is

$$W_{ij} = a_{ij} + a_{ji}\ \pi_i/\pi_j, \quad i \neq j\ . \quad (36)$$

If, as we suppose here, each country maintains a multilateral trade balance at all points in time, we have then

$L_i(t)\ \Sigma P_j(t) c_{ij}(t) = \Sigma P_i(t) L_j(t) c_{ji}(t)$ $i \neq j$ π_i is a function of $\hat{a}_{ij} = \frac{aijQi}{[1+tij]}$ (37)

Where t_{ij} is country i's tariff on imports from country j, $Qi{:}\ output$.

Taking into account country i's dynamic behavior, the specification of equation 26 gives X*(t) = Φ.

X(t). (38)

Where $X^*(t) = X_1(t), \ldots, X_j(t)$ and

$$\Phi = \begin{pmatrix} \Phi - \delta_X\ \Phi\ a_{12} w_{12} & \ldots & \Phi a_{1j} w_{1j} \\ . & & . \\ . & & . \\ . & & . \\ \Phi a_{j1} w_{j1} \Phi a_{j2} w j_2 & \ldots & \Phi - \delta_X \end{pmatrix}$$

The study of the international leveling out of the prices of goods and factors enables better understanding of cross-country volatility mechanisms.

The world has multiple countries; therefore we can consider multiple interferences. In this case, if radius are $R_0\ R_1\ R_2 \ldots.\ R_p \ldots.$ with an income amplitude τ^2 $\tau^2 p^2$ $\tau^2 p^4$ .....$\tau^2 p^{2p}$ …. and the phases are 0 $\Phi+2f_r$ $2\Phi+4f_r$ …. $p\Phi+2pf_r$ ….

T

Induced amplitude is, $A = \tau^2 + \tau^2 p + \tau^2 p^2 e^{-j(\Phi+2fr)+}$ $\tau^2 p\ ^4 e^{-j2((\Phi+2fr)}$

$$\ldots\ +\tau^2 p^2 e^{-j} p^{2(\Phi+2fr)} + \ldots \quad (39)$$

$$= \frac{\tau^2}{1 - p^2 e - j\ \Phi'} \quad (40)$$

$$\Phi' = \Phi + 2f_r \quad (41)$$

3.3.1.2 The intergenerational trade description

Our world has overlapping generations (or intergenerational trade) with no international trade; therefore each country operates under autarkical conditions. Each generation has initial endowments (at the beginning of the analysis) composed of natural and unnatural resources. Natural resources (the physical environment) and unnatural resources (all other resources) are the productive factors of the economy. Each generation has its own comparative advantages. Intergenerational trade is based exclusively on the productive factors and technology, hence, technology is considered here as a productive factor and its production depends only on the willingness of current generation to hoard down natural resources. The techniques production function $T(t) = G(\rho, E(t), N(t))$ is neoclassical with the following properties:

g(.) exhibits a constant return to scale, that is $G(\lambda E, \lambda N) = \lambda\ G(E, N)$, a property that is also known as homogeneity of degree one in E and N.

Positive and diminishing return to input:

$$\partial G/\partial E > 0\ \ \partial^2 G/\partial E^2 < 0 \quad (42)$$

$\partial G/\partial N>0 \quad \partial^2G/\partial N^2<0$ (43)

Inada conditions

$$\lim_{e\to 0}\frac{\partial G}{\partial E} = \lim_{n\to 0}\frac{\partial G}{\partial N} = \infty \quad (44)$$

$$\lim_{E\to\infty}\frac{\partial G}{\partial E} = \lim_{n\to\infty}\frac{\partial G}{\partial N} = 0 \quad (45)$$

$e^{-\rho t}$: is a generation's rate of time preference

Let us consider two generations in a given country with the current generation represented by (G_c) and the future generation represented by (G_f). The two generations are separated by a significant period of time so ordinary **tradable** goods cannot be stored. The two generations have a national status, thus we have successive nations in the same country. Each generation or nation has different initial endowments which are interdependent. If we suppose that all the generations of the country are co-owners of the country's resources, estimated as y'_i. Further, if each generation's life expectancy at birth is 100 years, the country's life expectancy at birth is 100n years for n generations. Each generation's initial endowment equals y_i'/n. Each country has n finite generations, 1, 2, …., n. $Y' = \Sigma y'_i$, Y' is intergenerational income and y'_i is a generation's Gross Domestic Product (GDP).

During the first generation's lifetime it uses its' y_i'/n of natural resources and borrows natural resources from following (coming) generations in different proportions(generation i's investment= I_i). Hence, the first generation's total natural resources, at the beginning of the first period, equals

$$\frac{\Delta y'i}{n} + \Sigma S'_{j1} \quad (46)$$

$\Sigma S'_{ij}$ is the first generation's debt, borrowed from the following generations (imported from the following generations). The second generation's total resources at the start of the second period is given as:

$$\frac{\Delta y'i}{n} - S_{21} + k_{12} + \cdots + \Sigma S_{j2}) \quad (47)$$

k_{12} represents the unnatural resources reimbursed from the first generation to the second generation. k_{12} should equal S_{21}. k_{12} represents the first generation's exports to the second generation and S_{21} is the first generation's imports from the second generation. The final generation's total resources equal

$$\frac{\Delta y'i}{n} - \Sigma S_{ni} + \Sigma k_{in} = \frac{S}{n} = S + K_n \quad (48)$$

The first generation uses its total natural resources to build the country (roads, schools, hospitals, airports, capital, research and development) and to produce goods and services for its own consumption. At the end of 100 years, the second generation, and those following, will have in co-ownership,

$$\Delta y'i - \Delta y'i[\beta + \delta(1-\beta)] \quad (49)$$

β is the self-consumption ratio (consumption by income units); δ is the ratio of remaining natural and unnatural resources (the portion of resources to be reimbursed to coming generations).

At the beginning of year 101, of this country's existence, the remaining resources are $\Delta y'i[(1 - \beta 1 - \delta$ (50)

The second generation's natural and unnatural resources are $\Delta y'i[(1-\beta)(1-\delta)]$. This generation proceeds like the first generation and at the end of its lifetime, the remaining resources are given by the following relationship

$\Delta y'i[(1-\beta)(1-\delta)] - \Delta y'i[(1-\beta)(1-\delta)][\beta + \delta(1-\beta)] = \Delta y'i[(1-\beta)(1-\delta)]^2$ These are third generation's resources.

At the start of the year 201, of this country's existence, the remaining resources are

$$\Delta y'i[(1-\beta)(1-\delta)]^2 \quad (51)$$

We notice the new resources follow a law of geometric progression, with (1-β) (1-δ) as the gain. The new resources of the n^{th} generation are $\Delta y'i[(1-\beta)(1-\delta)]^{n-1}$. (52)

The total amount of new resources equals the sum of the geometric progression with a gain less than one. This sum allows this limit, with the following expression:

$$\frac{\Delta y'i}{\Delta y'i[(1-\beta)(1-\delta)]} = \frac{\Delta y'i}{[\beta+\delta(1-\beta)]} = \Delta Y' \quad (53)$$

The optimal growth multiplier is $\frac{1}{[\beta+\delta(1-\beta)]}$. (54)

Hence, each wave of exchanges generates income fluxes across generations, following sinusoidal functions as $\Delta yit = y'i0\cos(W'ijt - \varphi 2)$. (55)

$$\Delta Y't = \Sigma\ \Delta y'it \tag{56}$$

Periodic function studies indicate each periodic movement with P as the period, is a sum sinusoidal movement with p, $\frac{p}{2}$, $\frac{p}{3}$, … as the periods. These represent the harmonics of the system.

$W'_{ij}(t)$ is the ratio of generation i's total trade with generation j (that is, generation i's bilateral exports and imports divided by generation i's output aggregate) represented as

$$W'ij \frac{\frac{Pj(t)}{Pi(t)}\ L'i(t)gij(t)+L'j(t)gji(t)}{Li(t)yi(t)} \quad i \neq j \ .(57)$$

gij (t) represents generation i's real per capita consumption of generation j's factors. P_i (t) is the price of factor i, and L'_i (t) is generation i's population, at each time period, t.

We now define a_{ij} (where $0 \le a_{ij} \le 1$) as a constant, representing a share of generation j's accessible natural resources which can be consumed by generation i as a part of their own unnatural resources. According to Abramovitz's social capability (1986), a_{ij} determines a generation's potential to adopt existing technologies. Using these definitions, the accumulated unnatural resources in generation i may be written as

$$X'^{*}_{i}(t) = \Phi\ [\Sigma a_{ij} w_{ij}(t)\ Xj(t)] + (\Phi - \delta_X)\ Xi(t)\ . \tag{58}$$

Where Φ represents the common productivity parameter and δ_X is the rate of depreciation of unnatural resource stock (obsolete or otherwise), assuming that $\Phi \ge \delta_X > 0$.

The measure of generation G_i's exchange with generation G_j, W_{ij} is

$$W_{ij} = a_{ij} + a_{ji}\ \pi_i/\pi_j, \quad i \neq j. \tag{59}$$

Supposing (supposed) as we do here that each generation maintains a multilateral trade balance at each point in time, we have

$L_i(t)\Sigma P_j(t)cij\ (t) = \Sigma P_i\ (t)\ Lj\ (t)\ cji\ (t) \quad i \neq j$ π_i is a function of $\hat{a}_{ij} = \frac{aijQi}{[1+tij]}$ (60)

Where t_{ij} is generation i's tariff on imports from generation j and Qi: $output$.

Taking into account generation i's dynamic behavior, the specification of equation 59 gives $X^*(t) = \Phi.\ X\ (t)$

where $X^*(t) = X_1(t), X_j\ (t)$ and

$$\Phi = \begin{pmatrix} \Phi - \delta_X & \Phi\ a_{12}w_{12} & \dots & \Phi a_{1j}w_{1j} \\ . & & & . \\ . & & & . \\ . & . & & \\ \Phi a_{j1}w_{j1} & \Phi a_{j2}w_{j2} & \dots & \Phi - \delta_X \end{pmatrix}$$

Each new generation of consumer-workers is born in the second half of the previous generation, in each country and lives for 100 years (generation t € [t-50, t+50]). Generation t exchanges nondurable and durable goods with generation t+1 but only durable goods with generations t+2, t+3 and onwards. Each of these generations has a finite number of consumers. Each consumer is endowed with one unit of labor and natural resource, supplied inelastically. The consumer can accumulate or save unnatural resources.

The sensitivity of intergenerational inter-dependencies can be analyzed as the effectiveness of intergenerational free exchange, and the extent to which that exchange affects prices in each generation. Describing the intergenerational exchange enables appreciation of price changes and their intergenerational transmission.

Natural resources, at the beginning, are divided equally among n generations. The remaining unnatural resources are the property of preceding generations. This could be viewed (regarded) as compensation for the natural resources used by one (a) generation (hoard down), but belonging to the following generations. It becomes (is) clear that each generation consumes part of the following generations' resources, reimbursing for that consumption with the remaining unnatural resources. This indicates that there is a clear trade between generations for the productive factors. Goods and services are indirectly exchanged through factor trade. This process of substitution enables us to postulate a transformation curve or the PPF for each generation along with its autarky prices or comparative advantages. Each generation has its own endowment of natural and unnatural resources. It is possible for a generation to make an arbitrage decision between the resources to export and those to import. If a generation chooses to consume more natural resources (imports) it therefore accepts having to produce more unnatural resources for coming generations (exports), and vice versa. According to the generation's demand for each good and service, we will have different comparative advantages. Each generation is then considered a different nation exchanging with other nations. If we consider two productive factors (natural and unnatural resources), two generations (G_c and G_f) and two goods (wheat and DVDs), there is a substitution process of the productive factors between generations. Following generations lend to preceding generations, their part of natural resources, receiving in return the remaining unnatural resources abandoned by the first generation at the end of their lives. The preceding and following generations indirectly exchange goods and services. The following generations indirectly sell goods and services to the preceding generations. These goods and services would have been produced with the following generations' allocation of natural resources if the following generation could appear during the preceding generations' lives to exchange the goods and services the preceding generations would have produced, with their remaining unnatural resources, in the periods of the following generations, if they could live during that future time. Therefore, the neoclassical models of international exchange can be applied to intergenerational trade as follows. Productive factors that exist in abundance in a generation and that are not intensively used to produce goods and services in that generation are exported to other generations in exchange for scarce productive factors intensively used to produce goods and services that should be scarce in the generation. The goods and services with weak consumption are indirectly exported from one generation to others, whereas goods and services with high consumption are indirectly imported from other generations. Thus, positive externalities (unnatural resources) are exchanged against negative externalities (overconsumption of natural resources). This externalities trade tends to equalize prices between generations. The Following generations would have an abundance of goods and services that use natural resources intensively. This would be possible if during their lives they can simultaneously have as many natural resources as possible along with the current abundant unnatural resources. Similarly, the current generation should have an abundance of goods and services that intensively use unnatural resources. This would be possible if they can have at their disposal as many of the following generations' additional abundant natural resources. Essentially, exports and imports represent intergenerational trade. For example, following generations sell natural resources with intensive wheat production values, or indirectly sell wheat to the current generation in exchange for unnatural resources intensive in DVD production. This exchange is made at the end of their lives or indirectly through DVDs. Although the DVDs did not exist during the period of the previous generation, this generation indirectly sold DVDs to the current generation by providing them with the technology inputs or knowledge necessary for DVD production (positive externalities).

Our hypothesis contradicts the neoclassical international trade model. We propose that only the productive factors are tradable. Final goods cannot be stored. To illustrate our intergenerational exchange model, we consider the Edgeworth box.

The beginning allocation is ω and the final is noted at point X. At point X a perfect equilibrium of production and consumption for the two generations is realized. Each generation improves its utility when passing from the lower indifference curve to the upper one. At that point, the quantities of produced and consumed goods, by all the generations (by pairs of two), are determined.

3.3.1.3 The multidimensional trade

Description: Each generation in a country is a seat (set) of sinusoidal movement (intergenerational movement effects). These movements can vary through different countries. For simplicity we assume, in this instance, that moments are the same, therefore cosine $(2\pi W_{ijt})e^{-t/\tau}$ is their most appropriate estimate. World income distribution is the movements' environment, which is supposed to be homogenous. W_{ij} is the period of time when the initial transaction impacts on countries revenue, during a group of processes. W_{ijt} represents the exchange for each group of processes. W_{ijt} is defined in equation 57.

$P_i(t) = \sum_{i=1}^{m} xipi$, x_i is the share of merchandise i within the value of total exports during the base year and pi is the current merchandise ratio price during the base year.

$P_j(t) = \sum_{i=1}^{m} mipi$, m_i is the share of merchandise i within the value of total imports during the base year and pi is the current merchandise ratio price during the base year.

W'_{ij} is the number of times the initial movement impacts on generations during a group of processes. W'_{ijt} represents the exchange of value for each group of processes. W'_{ijt} is defined in equation 34.

$P'_i(t) = \sum_{i=1}^{m} x'ip'i$, x'_i is the share of merchandise i within the value of total exports for the base generation and p'i is the current merchandise ratio price for the base generation.

$P'_j(t) = \sum_{i=1}^{m} m'ip'i$, m'_i is the share of merchandise i within the value of total imports for the base generation and p'i is the current merchandise ratio price for the base generation.

The production function is

$$Y_r = AE^{\alpha}N^{\beta}X^{*}i(t).\exp(\varepsilon i, t)\ . \qquad (62)$$

Y_r is increasing, concave, continuously differentiable and homogenous of degree one.

Producers minimize their costs, taking given prices and earn no profit.

Consumers in each country and generation maximize their utility, as stated above.

We now consider τ as the time period of an intra-industrial transaction (W_{ij}). This transaction (W_{ij}) generates a sinusoidal impact on world current income. W'_{ij} is an intergenerational movement and τ' is its time period. This transaction (W_{ij}) generates a sinusoidal impact on intergenerational incomes (the sum of all generations' incomes).

See Fig.1. Multidimensional trade description

And Graph 2. Multidimensional trade box: initial and final endowments and multidimensional trade equilibrium determination in Appendix.

The expression of Multidimensional trade

Building upon Grossman and Helpman's (1991b) proposition, $W_{ij}(t)$ is the ratio of country i's total trade (generation i') with country j (generation j'). That is, country i's (generation i') bilateral exports and imports are divided by country i's aggregate output (generation i').

$$Wij = \frac{\frac{Pj(t)}{Pi(t)}\ Li(t)gij(t)+Lj(t)gji(t)}{Li(t)yi(t)} \qquad i \neq j$$

$$W'ij = \frac{\frac{P'j(t)}{P'i(t)}\ L''i(t)g'ij(t)+L''j(t)g'ji(t)}{L'i(t)y'i(t)} \qquad i' \neq j'$$

If these two flows have the same rhythm, but different country (generation) weights, the macro-dynamic equilibrium, or multidimensional trade, represents interference between the international transaction (W_{ij}) and the intergenerational transaction (W'_{ij}). These two situations are described above.

$$\Delta Yt = \Sigma \Delta yit + \Delta yit$$

$$= yi0\cos(Wijt - \varphi_1) + y'i0\cos(W'ijt) - \varphi_2) \qquad (63)$$

If we develop equation 63, we obtain:

$$\Delta Y_O \cos t \cos\varphi + \Delta Y_O \sin \mathrm{Wijt} \sin\varphi = y_{i0}\cos \mathrm{Wijt} \cos\varphi_1 + y_{i0}\sin \mathrm{Wijt} \sin\varphi_1 + y'_{i0}\cos \mathrm{Wijt} \cos\varphi_2 + y'_{i0}\sin \mathrm{Wijt} \sin\varphi_2 \qquad (64)$$

Solving simultaneously:

$$\Delta Y_O \cos \mathrm{Wijt} \cos\varphi = y_{i0}\cos \mathrm{Wijt} \cos\varphi_1 + y'_{i0}\cos \mathrm{Wijt} \cos\varphi_2 \qquad (65)$$

$$\Delta Y_O \sin \mathrm{Wijt} \sin\varphi = y_{i0}\sin \mathrm{Wijt} \sin\varphi + y'_{i0}\sin \mathrm{Wijt} \sin\varphi_2 \varphi_2 \qquad (66)$$

This becomes:

$$\Delta Y_O \cos\varphi = y_{i0}\cos\varphi_1 + y'_{i0}\cos\varphi_2 \qquad (67)$$

$$\Delta Y_O \sin\varphi = y_{i0}\sin\varphi_1 + y'_{i0}\sin\varphi_2 \qquad .(68)$$

We then calculate the amplitude of multi-dimensional trade as:

$$\Delta Y_O{}^2(\cos^2\varphi + \sin^2\varphi) = y_{i_o}^2(\cos^2\varphi_1 + \sin^2\varphi_2) + y'^2_{i_o}(\cos^2\varphi_1 + \sin^2\varphi_2) + 2y_{i_o}y'_{i_o}(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2 \qquad (69)$$

$$\Delta Y_O{}^2 = y_{i_o}^2 + y'^2_{i_o} + 2y_{i0_o}y'_{i_o}\cos(\varphi_1 - \varphi_2) \qquad .(70)$$

If multidimensional trade is horizontal ($\varphi_1 = \varphi_2$),

we have $\Delta Y_O{}^2 = y_{i_o}^2 + y'^2_{i_o}$. (71).

In this case we have constructive multidimensional trade because the trade increases.

If multidimensional trade is vertical, with different generational weightings **(**$\varphi_1 = \varphi_2 + \pi$), we obtain $\Delta Y_O{}^2 = y_{i_o}^2 - y'^2_{i_o}$. (72)

In this situation multidimensional trade is destructive as it decreases.

Between these two extremes, multidimensional trade varies with the cosine (φ_1- φ_2) or the cosine of different generational weightings.

A generation's weight is calculated by dividing the preceding equations, member by member, as follows

$$Tan\varphi = \frac{y_{i_O}\ sin\,\varphi_1 + y'_{i_O}\ sin\,\varphi_2}{y_{i_O}\ cos\,\varphi_1 + y'_{i_O}\ cos\,\varphi_2} \quad (73)$$

Finally, multidimensional trade is expressed as

$$\Delta Y_O{}^2 = y_{i_O}^2 + y'^2_{i_O} + 2y_{i_O}y'_{i_O}\ cos(\varphi_1 - \varphi_2)\ cos\left(\text{Wijt} - arctan\varphi\frac{y_{i_O}\ sin\,\varphi_1 + y'_{i_O}\ sin\,\varphi_2}{y_{i_O}\ cos\,\varphi_1 + y'_{i_O}\ cos\,\varphi_2}\right) .(74)$$

With the Fourier transform we obtain spectral frequencies like

$$\text{F(Wijt)} = \int f(t)\,e2\pi jWtdt = \frac{y_{i_O}}{[2]}\int [e2\pi j(Wij0 + Wij)t + e2\pi j(Wij0 - Wij)t]dt$$

$$= \frac{y_{i_O}}{[2]}\frac{1}{\left[\frac{1}{\tau} - 2\pi j(wij0+wij)\right]} + \frac{y_{i_O}}{[2]}\frac{1}{\left[\left[\frac{1}{\tau} - 2\pi j(wij0+wij)\right]\right]} \quad (75)$$

$$[F(wij)]^2 = \frac{1}{\left[\frac{1}{\tau^2} + 4\pi^2 j(Wij0+Wij)^2\right]} \qquad \Delta Wij = \frac{1}{[2\pi\tau]}.$$

Derived consumption function

Each generation maximizes its overall utility according to its time of life as given by

$$U_{gi} = \max \sum_{t=0}^{n} \binom{n}{k} ß^t u(ci1t, ci2t) = \int_0^\infty \frac{\Sigma\Delta yit}{[\beta+\delta(1-\beta)]} e^{nt} e^{-\rho t}dt = \int_0^\infty u(c)\,e^{nt}e^{-\rho t}dt \quad (76)$$

$$with\ u(c) = \frac{\Sigma\Delta yit}{[\beta + \delta(1-\beta)]}$$

s.t. $p_b c_{id} + p_{dt} c_{idt} + w_i r x_{it} + r_{it} + \partial \leq w_{it} + r_{it} k_{it} + (\partial + r_{it}) r_{it}$

$k_{it} + \partial - (1-\delta)k_{it} \leq x_{it}$

$c_{ijt} \geq 0$, $x_{it} \geq 0$, $b_{it} \geq -B$

$k_{i0} \leq k_{-i0}$, $b_{i0} \leq 0$

$e^{-\rho t}$: is a generation's rate of time preference

If we pose: a as asset per person; r: interest rate; w is the wage rate and n is the growth rate of population

these constraints can be resumed as

$\dot{a}$ = (r-n).a + w –c (see Barro and al. 2004).

with

$$\Delta Y_{Ot} = \frac{y_{i_O}}{[2]}\frac{1}{\left[\frac{1}{\tau} - 2\pi j(wij0+wij)\right]} + \frac{y_{i_O}}{[2]}\frac{1}{\left[\left[\frac{1}{\tau} - 2\pi j(wij0+wij)\right]\right]} = AE^{\alpha}N^{\beta}X^*_i(t).\exp(\varepsilon i,t) + AE'^{\alpha'}N'^{\beta'}X'^*_i(t).\exp(\varepsilon i',t) + \sqrt{f(.)} \quad (77)$$

$$f(.) = \frac{1}{\left[\frac{1}{\tau^2} + 4\pi^2 j(Wij0+Wij)^2\right]}$$

and $wij = \dfrac{\frac{Pj(t)}{Pi(t)}\ Li(t)gij(t) + Lj(t)gji(t)}{Li(t)yi(t)} \quad i \neq j$

That is, generation's utility at time 0 is a weighted sum of all contemporaneous consumptions utilities, u(c). We assume that u(c) is increasing in c and convex, u'(c)<0, u''(c) >0. The convexity describes an individual overall satisfaction over time as he tends to the end of his life. At the end of a generation's life, all non-durable goods are consumed and the unnatural durable resources - include the level of technology- survive as a payment of its overconsumption of natural resources.

The individual utility u(c) has been multiplied by the generation size, L = e^{nt} showing the adding up of utils for all generation members alive at time t. $e^{-\rho t}$ - with ρ) exhibits time preference's rate, describing the fact that generation t-1' s preference to consume at time t-1 than t and its reimbursement to generation t should include interests.

A point of time utility function is homothetic, strictly increasing, strictly concave, and continuously differentiable.

The first order conditions of the utility function are:

$$\frac{ud(cibt, cidt)}{Ub((cibt, cibt)} > \frac{Pdt}{Pbt}$$

$$\frac{Ub(cibt,cidt)}{ßUb((cibt+1,cidt+1)} > \frac{Pbt}{Pbt+1}(\text{wit}+1)(1-\partial) + \text{rit} + 1$$

if $q^i_t > 0$ (78)

$$1 + rbi(t+1) \geq \frac{wi(t+1)(1-\partial)+ri(t+1)}{wit}, = \text{if } q^i_t > 0 \quad (79)$$

Consumption function in Ramsey model (see Barro and al(2004) is given by

$$C(t) = c(0).e^{1/\theta/[\bar{r}(t)-\rho]t} \quad (80)$$

The substitution of this result for c(t) into the intertemporal budget constraint in equation (8) leads to the consumption function at time 0:

$c(0) = \mu(0).\ [a(0) + \bar{w}(0)]$

Where μ(0), the propensity to consume out of wealth, is determined from

$$[1/\mu(0).] = \int_0^{\infty} e^{\bar{r}(t)\left(1-\theta\right)/\theta-\frac{\rho}{\theta}+n\big]t}\,dt \qquad (81)$$

Derived production function

Considering the multidimensional trade expression:

$$\Delta Y_{O,t}{}^2 = y_{i_O}^2 + y'^{\,2}_{i_O} + \frac{1}{\left[\frac{1}{\tau^2}+4\pi^2 j(wij0+wij)^2\right]} \qquad (82)$$

And combining equations 82, 11 and 67:

$Y_{it} = AE^{\alpha}N^{\beta}X^*_i(t).\ \exp(\varepsilon i,t)$ (see equation 11)

$y'_{i_O} = AE'^{\alpha'}N'^{\beta'}X'^*_i(t).\ \exp(\varepsilon i',t)$ (see equation 67)

We obtain:

$$\Delta Y_{Ot} = AE^{\alpha}N^{\beta}X^*_i(t).\exp(\varepsilon i,t)+AE'^{\alpha'}N'^{\beta'}X'^*_i(t).\exp(\varepsilon i',t)+\sqrt{f(.)} \qquad (83)$$

$$f(.) = \frac{1}{\left[\frac{1}{\tau^2}+4\pi^2 j(Wij0+Wij)^2\right]}$$

with $wij = \dfrac{\frac{Pj(t)}{Pi(t)}\,Li(t)gij(t)+Lj(t)gji(t)}{Li(t)yi(t)} \qquad i \neq j$

The logarithm linear regression of equation 83 in per worker form can be expressed

$$\left(\frac{Y}{L}\right)i,t = \ln(A_i+A'_i)+(\alpha_E+\alpha'_E)\ln\left(\frac{E}{L}+\frac{E'}{L'}\right)+(\beta_N+\beta'_N)\ln\left(\frac{N}{L}+\frac{N'}{L'}\right)+[(a_{ij}W_{ij}(t)+a'_{ij}W'_{ij}(t)]\ [Xj(t)+X'j(t)]+\delta''_X X'i(t)$$

$$+(\alpha_E+\beta'_N+a_{ij}W_{ij+}\ {}_{\delta'X)\ln N}+\frac{1}{\left[\frac{1}{\tau}-2\pi j(wij0+wij)\right]} \qquad (84)$$

Equilibrium: The behavior of competitive households and firms in a generation interacting with households and firms of another generation has been completely described. The resulting equilibrium is multidimensional. This equilibrium is obtained through the international and intergenerational leveling out of goods and factors' prices.

International leveling out of goods and factors' prices

Um_{wheat}/ represents the wheat price while Um_{DVD}/ represents the price of DVDs.

The wheat price is shown as P_b and DVD prices are indicated by P_d.

Marginal utility is described by U_m.

The international equilibrium price is 2b/d (for example, two units of wheat to one DVD). This result indicates wheat prices have risen in China compared to the autarky, which was 3b/d (three units of wheat to one DVD).

The same international trade price indicates DVD prices fell in China. A symmetric adjustment will take place in the United States where P_b decreases and P_d augments. In China, wheat production augments and DVD production decreases. Natural resource demand will increase causing price rises. Proportionally, the natural resources in wheat production will decrease while the proportion of unnatural resources in wheat production will increase. In China, the changing factor prices will modify production techniques. The techniques will intensify unnatural resources. In the United States the reverse will be the case; techniques will be intensive in natural resources with prices decreasing.

Therefore, in China, wage rates augment while in the United States wage rates decrease. The general international equilibrium will have all prices leveling out because changes are the symmetrical reverse from one country to another.

The first order conditions for profit maximization are:

$$P_b \geq (w+r)f_b(q_b,\ q_d),\ \text{if } q_b>0 \qquad (86)$$

$$P_d \geq (w+r)f_d(q_b,\ q_d),\ \text{if } q_d>0\ . \qquad (87)$$

For the production functions with constant output, the minimum cost is a linear function of π, of $\varphi_{tf,}$ π depends on w et r.

Then,

$$C_{usd}(w,r,Q_{usd}) = \pi.Q_{usd} \text{ and } \pi = \pi f(w,r)r) \qquad (88)$$

$P_{usd} = \frac{\partial C_{at}}{\partial Q_{usd}} = \pi_t(w,r)$ for the DVDs and (89)

$P_{usb} = \pi_{us}(w,r)$ for the wheat,

$r = r(P_{usd}, P_{usb})$ b and $w = w(P_{usd}, P_{usb})$ where $\frac{w}{r} = h\left(\frac{P_{usb}}{P_{usd}}\right)$. (90)

The relationship within the two countries is identical. The price of goods and services is leveling out as are the factor prices in all countries. We conclude that there is a convergence towards a constant rate of equilibrium growth, where the stocks of unnatural and natural resources are superior to their equilibrium level.

Intergenerational leveling out of goods and factors' prices

At the intergenerational equilibrium the following relations are identified:

Um_{wheat}/ wheat price = Um_{DVD}/ DVD price.

The intergenerational trade equilibrium can also be represented through a system of iso-product curves for each good as a dual program.

For example, the current French generation is well endowed in unnatural resources and with the following generations' natural resources. At the beginning of intergenerational trade, 'current French' will export unnatural resources (indirectly the DVDs, a product with intensively high unnatural resources) and will import natural resources (indirectly the wheat, a product with a high proportion of natural resources) from the 'future French' with an intergenerational equilibrium price of 3r/t. This result indicates the price for unnatural resources has been augmented compared with the autarky price, which was 2r/t.

The same intergenerational trade price shows the price for natural resources has reduced for the 'current French'. A symmetrical adjustment will take place with the 'future French', when P_t decreases and P_r augments. For the 'current French', the proportion of natural resources in wheat production will increase while the proportion of unnatural resources decreases. For the 'current French', the change in the factor prices will modify production techniques. Techniques will use more natural and less unnatural resources. For the 'future French', the reverse applies; techniques will be intensive in unnatural resources and their prices will fall. The substitution of natural resources for unnatural resources in wheat production causes wheat prices to fall for the 'current French'. A symmetric analysis indicates DVD prices will decrease and wheat prices will rise for the 'future French'. Therefore, for the 'current French', $\frac{P_{\mathrm{w}}}{P_{\mathrm{d}}}$ augments and for the 'future French', $\frac{P_{\mathrm{w}}}{P_{\mathrm{d}}}$ decreases. At the general intergenerational equilibrium, all prices will level out because their changes are the symmetrical reverse from one period to another. Intergenerational trade productive factors reduce the prices of rare factors in each period and enable the production of goods and services consumed in a particular period. The lower prices of goods and services in a particular period cause intergenerational trade earnings for consumers and producers of the given period.

For the production functions with constant outputs, the minimum cost is a linear function of □□□of □$_{tf,}$□□depending on w and r.

$MinC_r = wE_r + rN_r$ (91)

subject to

$Y_r = AE^{\alpha}N^{ß} X^*_i(t) \exp(\varepsilon i, t)$.

For example, iso-product unit curves and iso-cost curves can be established. This program's solution enables us to determine the optimal production corresponding to the minimum cost. This equilibrium is obtained at the tangency point of the iso-product unit curve and the lowest possible iso-cost curve. This point gives the leveling out of the intergenerational terms of trade and the equivalency of the values of the goods and the factors exchanged

Then,

$C_{usd}(w,r,Q_{usd}) = \pi . Q_{usd}$ and $\pi = \pi f(w,r)$r) (92)

$P_{usd} = \frac{\partial C_{at}}{\partial Q_{usd}} = \pi_t(w,r)$ for the DVDs and (93)

$P_{usb} = \pi_{us}(w,r)$ for the wheat,

$r = r(P_{usd}, P_{usb})$ b and $w = w(P_{usd}, P_{usb})$ where $\frac{w}{r} = h\left(\frac{P_{usb}}{P_{usd}}\right)$. (94)

The relationship within the two countries is identical. The price of goods and services is leveling out as are the factor prices in all countries. We conclude there is a convergence towards a constant rate of equilibrium growth, where the stocks of unnatural and natural resources are superior to their equilibrium level.

The steady state

We now have necessary tools to analyze the behavior of the model over time. We first consider the long run or steady state, and then we describe the short run or transitional dynamics. The steady state is generally described as a situation in which the various quantities grow at constant rates. In the traditional model of Solow-Swan, the steady state is found at an intersection of s.f(k) curve and $(n + \delta_X)k$, the depreciation line.

This production function can be rewritten as:

$$Y(t) = F[N(t), E(t), T(t)] \quad (95)$$

N(t), the unnatural Input, E(t), natural input and T(t), the level of technology which is assumed to be determined by consumption level. At this level, we still maintain neoclassical assumption that technology is freely available within a generation to all firms but, for this analyze, is fully excludable between generations.

If we pose K= N(t). E(t), we obtain AK model where A or T(t) is a positive constant that reflects the level of the technology. If we substitute $f(\eta)/\eta = A$ in $\eta = s.f(\eta) - (n+\delta).\eta$

We get $$\dot{\eta}/\eta = s.A - (n+\delta). \quad (96)$$

We see that s.A and (n+ δ) are the horizontal lines and, hence $\dot{\eta}/\eta$ is the vertical distance between the two lines. Therefore $\dot{\eta}/\eta$ is a constant and independent of η ; that is η continues to grow at the steady state rate $(\dot{\eta}/\eta)^* = Sa - (n+\delta)$. It is clear that $y = A\eta$, $\dot{y}/y = \dot{\eta}/\eta$ at every point of time. Since c= (1-s) . y, $\dot{c}/c = \dot{\eta}/\eta$. We see that all per capita variables in the model will permanently grow at the same rate sA –(n+ δ) . considering that a generation that increases its consumption of natural resources (overconsumption) and hence his physical capital, learns simultaneously how to produce efficiently and will reimburse to future generations a great level of technology (unnatural resources).

$$\delta = \frac{\partial gij(t) + \partial' gji(t)}{yi(t)} \quad (97)$$

In this model, the net increase in the stock of unnatural resources at a point of time equals gross investment less depreciation:

$X'^*_i(t) = \Phi\,[\Sigma a_{ij} w_{ij}(t)\, Xj(t)] + (\Phi - \delta_X)\, Xi(t)$ corresponds to $\eta = d(N/L)/dt = N/L - n\eta$

In Solow-Swan model

And at a point of space (country level)

$X^*_i(t) = \Phi[\Sigma a_{ij} w_{ij}(t) X_j(t)] + (\Phi - \delta_X) X_i(t)$ also corresponds to $\eta = d(N/L)/dt = N/L - n\eta$

In Solow-Swan model

If we state: Ĺ/L= n : population natural growth rate. If s is the saving rate, we have:

$$N/L = s.\ [\ln(A_i + A'_i) + (\alpha_E + \alpha'_E)\ln(\frac{E}{L} + \frac{E'}{L'}) + (\beta_N + \beta'_N)\ln(\frac{N}{L} + \frac{N'}{L'}) + [(a_{ij}W_{ij}(t) + a'_{ij}W'_{ij}(t)]\ [Xj(t) + X'j(t)] + \delta''_X X'i(t)$$

$$+(\alpha_E + \beta'_N + a_{ij}W_{ij+}\ {}_{\delta'X)\ln N} + \frac{1}{\left[\frac{1}{\tau} - 2\pi j(wij0 + wij)\right]}] - \delta\eta = s.\ f(\eta) - \delta\eta \quad (98)$$

$$\eta = s.f(\eta) - (n+\delta).\ \eta \quad (99)$$

If a generation expands N_i, then K rises in parallel and increase the productivity of the following generations. The marginal product of K should equal the intergenerational interest rate and $I_{gc} = S_{gf}$

The saving rate is determined by the first generations which decide what quantities of natural resources belonging to future generations to invest in production. This overconsumption of natural resources constitutes current generation investment and a debt to pay to the next generations in terms of unnatural resources. The more a current generation overconsumes in terms of natural resources, and hence it consumes high level of goods, the more it will invest in R&D and should have a great impact on technology that will use the next generations. In general, $I_{gc} = S_{gf}$ It is not possible to have $I_{gc} < S_{gf}$ or vice versa. I_{gc}: Investment of current generation, S_{gf} :Saving of future generation. The technological progress is decreasing over time. This assumption is based on the fact that the truth on everything is unique and when the truth is discovered the partial knowledge will disappear.

A generation's gain can be written

$$E_i.[F(\eta_i, K) - (n+\delta).\ \eta_i - w] \quad (100)$$

If we assume that each firm and consumer in a generation operates in a competitive world and takes each factors prices as given, K is also given. A generation zero-gain maximization conditions lead to

$\partial y_i/\partial \eta_i, = F_1(\eta_i,,K) = r + \delta)$ (101)

$\partial y_i/\partial E_i = F(\eta_i,, K)- \eta_i, . F_1(\eta_i,,K) = w$ (102)

The average product of unnatural resources can be written

$F(\eta_i, K)/ \eta_i, =f(K/ \eta_i) = f(E)$ (103)

This function of average product of capital satisfies f'(E)> and f''(E)<0. The spillover effects eliminate the tendency for diminishing returns.

The marginal product of capital derived from F(E) is

$F_1(\eta_i,, K) =f(E) -E.f'(E)$. This marginal product of capital is less than F(E) and do not depend on η. We see that since f''(E)< 0, the marginal product of unnatural resources is increasing in E.

Equilibrium

Considering the following equations

$\dot{a}$ = (r-n).a + w –c (104)

$\dot{c}/c = (1/ɵ). (r- \rho)$

Transversality condition $\lim_{n\to\infty}\{a(t).\exp[-\int_0^t[r(v)-n]dv]\} \geq 0$ (105)

and

$r =F_1(\eta, K) – \delta$, (106)

the marginal product of capital can be rewritten

$\dot{c}/c = (1/ɵ).[f(E) – E. f'(E) – \delta – \rho]$ (107)

The accumulation function for η is

$\dot{\eta} = f(E) . \eta – c – \delta \eta$ (108)

This model because of transversality condition has no transitional dynamics:

Since c= (1-s) . y, $\dot{c}/c = \dot{\eta}/\eta$. We see that all per capita variables in the model will permanently grow at the same rate (1/ɵ).[f(E) – E. f'(E) – δ – ρ]. (109)

The saving and investment increase among the first generations and decrease when we tend towards the end of the country.

F(.) satisfies the neoclassical properties.

If $\hat{L}$ =L.T(t), we have:*

$Y =F(N, \hat{L})$ (110)

If $\hat{y}= Y/\hat{L}$ and $\hat{\eta}= K/\hat{L}$ (111)

The production function becomes

$\hat{y}f(\hat{\eta})$ (112)

It is demonstrated that each firm that takes r and w as given maximizes profit for given □

By setting $f'(\hat{\eta}) = r +\delta$ (114)

At the equilibrium $\hat{\eta} = f(\hat{\eta}) – □ – (□ + □ + \delta). \hat{\eta}$ (115)

s.f(η)/N is a horizontal line at the level (1/ɵ).[f(E)

The transversality condition can be written:

$\lim_{□\to\infty}$ $\{\hat{\eta}.\exp(\int_0^{□}[□'(\hat{\eta}) - \delta - x - n]dv\}$(116)

When a country chooses production initially different from W, it should compensate overconsumption of natural resources by an equivalent measure of unnatural resources to establish, or maintain, constructive multidimensional trade. If not, the country and the world may experience volatility. This volatility varies according to the distance between effective trade production (W_i) and initial optimal trade production, along with the sensitivity of the international interdependencies. Therefore, the country's PPF is moving around the World Technology Frontier. Derived growth is not Pareto-optimal (see Graphs 1&2). The international volatility function is described as

$(X_f – X) = f(W_f – W, ɵ')$. (117)

ɵ' is the international sensitivity factor. Volatility becomes explosive (across other countries) if international interdependencies are very sensitive. Hsieh and Klenow (2009) and Klenow (2012) discuss this mater. They use micro data from manufacturing establishments to quantify and compare potential resource misallocations between the United States and India. Their research indicates resource misallocation can lower aggregate total factor productivity (TFP) and growth.

For the same reasons, when a generation initially chooses production different from W, this generation should compensate for its overconsumption by an equivalent measure of unnatural resources. This will maintain or establish constructive multidimensional trade. If this compensation is not made, the generation and the world potentially experience significant volatility. This volatility varies according to the distance between the effective trade production (W_i) and the optimal initial trade production, along with the sensitivity of the intergenerational interdependencies. Therefore, the generation's PPF moves around the World Technology Frontier. Derived growth is not Pareto-optimal (graphs 1&2). The intergenerational volatility function can be described by the following relationship

$$(X_f - X) = f(W_f - W, ɵ') \tag{118}$$

ɵ is the intergenerational interdependency sensitivity factor. Volatility becomes explosive (through other countries and generations) if the interdependencies are particularly sensitive.

Volatility drivers of markets (capital and goods) are prices and their associated flexibility.

See Graph 1: **Impacts on growth of World and Intergenerational PPF' s movements** in Appendix.

In the general case, prices and quantities adjustment process is widely depicted through international and intergenerational trade. The prices of goods and services are leveling out as are the factor prices in all countries. We conclude there is a convergence towards a constant rate of equilibrium growth, where the stocks of unnatural and natural resources are superior to their equilibrium level. At the general intergenerational equilibrium, all prices will level out because their changes are the symmetrical reverse from one period to another. Intergenerational trade productive factors reduce the prices of rare factors in each period and enable the production of goods and services consumed in a particular period. The lower prices of goods and services in a particular period cause intergenerational trade earnings for consumers and producers of the given period. As we can see, this general case is the rule but, many factors such as distortions on some markets (due to bad policies) put the production possibilities frontiers in a sort of movement in a way that the directions taken by these movements in each country and/or generation interact with international or intergenerational trade to determine long run per capita growth. The direction of these movements depends on how government intervention and other shocks impact productive resources allocation. The level of resources could rise or drop and the production technologies or the intergenerational marginal rate of substitution of resources could change. Even though only differences in the change of countries/generations' resources should lead to a change into the comparative advantages and international/ intergenerational trade configuration, these distortions should cause disturbance on the relationship between growth and economic volatility. The sign of the relationship between growth and volatility then should depend on these movements and their interaction with international and intergenerational trade. For King et al. (1988), a temporary disturbance to production possibilities frontiers can have permanent effects on the path of the output growth. The importance and the nature of these effects depend on the types of the disturbances.

4. CONCLUSION

In the Ak model, an improvement in the level of technology, A, which raises the marginal and average products of capital, also raises the growth rate and alters the saving rate. In contrast to the effects on long-run growth in the AK model, the Ramsey model implies that the long-run per capita growth rate is pegged at the value x, the exogenous rate of technological change. A greater willingness to save or an improvement in the level of technology shows up in the long-run as higher levels of capital and output per effective worker but in no change in per capita growth rate.

In the neoclassical model if diminishing returns set in slowly, shift in the willingness to save or the level of technology affect the growth rate for a long time. Therefore, the differences between the neoclassical and AK models depend on the speed of convergence to steady state.

The core result of our model is that greater willingness to hoard down or an improvement in the level of technology shows up in the long-run as higher levels of capital and output per effective worker to determine higher level in per capita growth rate. The steady state results of the working of diminishing returns to inputs in technology production function.

In fact, the more a current generation overconsumes in terms of natural resources (hoarding down), and hence it consumes high level of goods, the more it will invest in R&D and should have a great impact on technological progress, part of unnatural resources to sale to the following generations. The prices of goods and services are leveling out as are the factor prices in all countries. We conclude that there is a convergence towards a constant rate of equilibrium growth, where the stocks of unnatural and natural resources are superior to their equilibrium level. That is, intergenerational trade productive factors reduce the prices of rare factors in each period and enable the production of goods and services consumed in a particular period.

As we can see, this general case is the rule but, many factors such as distortions on some markets (due to bad policies) put the production possibilities frontiers in a sort of movement in a way that the directions taken by these movements in each country and/or generation interact with international or intergenerational trade to determine long run per capita growth. The direction of these movements depends on how government intervention and other shocks impact productive resources allocation.

In the multidimensional trade theory, the externalities trade enables to include in the model all intergenerational markets. Therefore, multidimensional trade model appears as the best linear unbiased externalities internalization (BLUEI). Subsequently, due to the simultaneity of cross-country and cross-generation links in the multidimensional trade, all Walrasian equilibria are Pareto-optimal.

In addition, multidimensional trade appears to have multiple movements which propagate vertically (through generations) and horizontally (through nations) inducing economic interferences. The study of the general equation of multidimensional trade (economic interferences) shows the existence of constructive, destructive and indeterminate trade and links between growth and volatility.

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Author has declared that no competing interests exist.

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APPENDIX

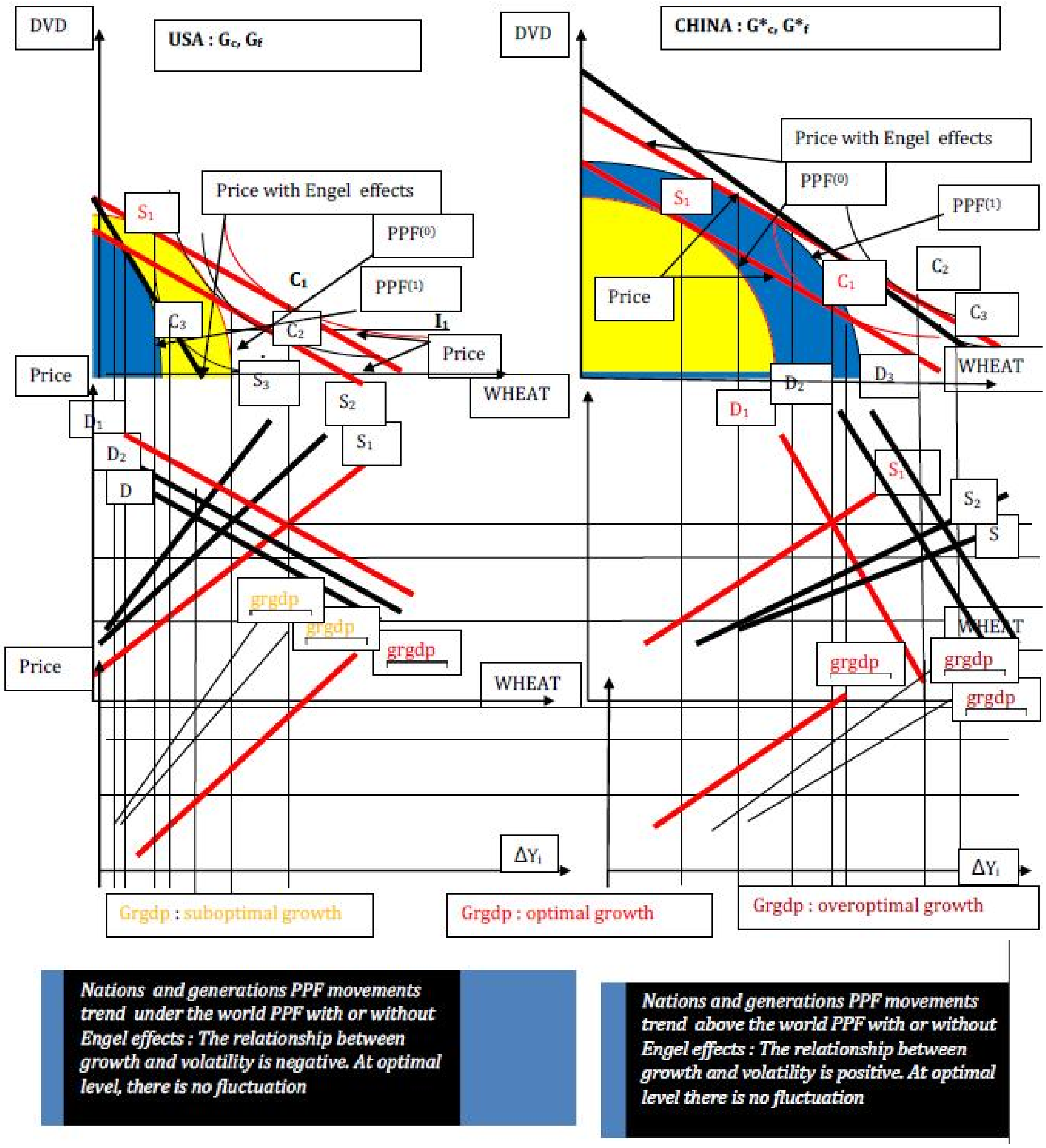
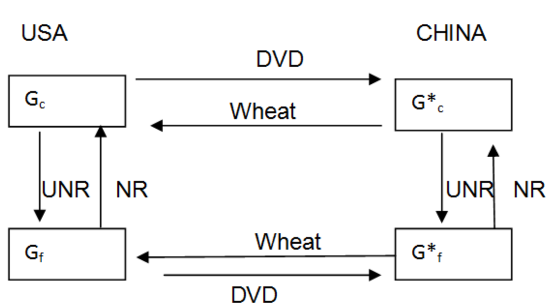
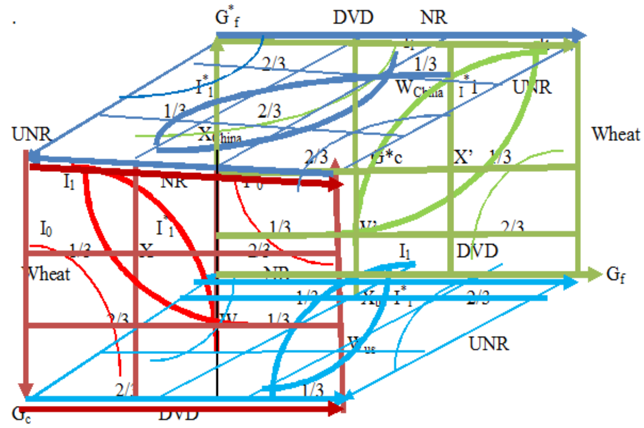


Fig. 1. Impacts on growth of World and Intergenerational PPF movements



Graph 1. Multidimensional trade description



NR : Natural resources ; UNR : Unnatural resources

I_i : Indifference curves.

On each face of the cube I describe trade between two generations of the same country (intergenerational trade) or between one generation of one country and one generation of another country (international trade):

The base of the cube (the base bleue box) describes trade between G_c and G_f. Wus(2/3UNR, 1/3NR) is the initial endowment of US current generation. Its final endowment is Xus(1/3UNR,2/3UR). The equilibrium between Gc and Gf is determined.

On the top bleue box the same trade happens between G^*_c and G^*_f of China. W_{China}(1/3NR,2/3UNR) and X_{China}(2/3NR,1/3UNR) are respectively G*c initial and final endowment and symmetric values for G^*_f, W_{China} (2/3 NR,1/3UNR) and X_{China}(1/3NR,2/3UNR).

The red box describes final goods' trade and equilibrium between G_c and G^*_c and green box describes final goods' trade and equilibrium between G_f and G^*_f.

Graph 2. Multidimensional trade box: Initial and final endowments and multidimensional trade equilibrium determination

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