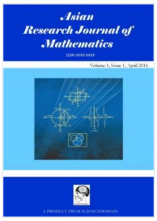

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Oscillatory Behavior of the Solutions for a Nonlinear Mechanical System with Delay

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, the oscillatory behavior of the solutions for a class of nonlinear mechanical system with delay is investigated. By means of mathematical analysis method, some sufficient conditions to guarantee the oscillation of the solution are obtained. Computer simulations are provided to demonstrate our results.

Keywords: Nonlinear mechanical system; delay; oscillation.

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1 Introduction

It is well known that stability and oscillations are two research topics for many mechanical systems [1-20]. In [1], Rabeloa et al. investigated a two-degree-of-freedom mechanical model with damping which subjects the time delay. This model consists of a primary system attached to the ground by a suspension that includes damping and spring, and a damped secondary mass coupled to

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the primary system by a spring with nonlinear characteristics. The mathematical model for this mechanical system is as follows:

$$\begin{cases} y_1'' + \omega_1^2 y_1 + \alpha_{12} y_1^2 + \alpha_{13} y_1^3 + \zeta_1 y_1' + \zeta_2 y_1' y_1^2 - \alpha_{21} y_2 + \zeta_3 (y_1'(t-\tau) - y_2'(t-\tau)) \\ \qquad + \alpha_{22}(y_1 - y_2)^2 + \alpha_{23}(y_1 - y_2)^3 = F_1 \cos(\Omega_1 t) + y_2 F_2 \cos(\Omega_2 t) \\ y_2''(t) + \omega_2^2 (y_2 - y_1) + \beta_{22}(y_2 - y_1)^2 + \beta_{23}(y_2 - y_1)^3 + \zeta_4 (y_2'(t-\tau) - y_1'(t-\tau)) = 0. \end{cases} \tag{1}$$

where ω_1, ω_2 are natural frequencies; Ω_1, Ω_2 represent the forcing frequencies; $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ represent the damping parameters; $\alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23}$ are stiffness parameters; F_1, F_2 represent parameters of the external excitation force amplitudes. Under the restricted conditions for small damping, for small amplitudes of external excitation and weak stiffness of nonlinearities, that is, for a small parameter of perturbation ε $(0 < \varepsilon \ll 1)$:

$$\alpha_{ij} = O(\varepsilon), i = 1,2, j = 1,2,3; F_i = O(\varepsilon), i = 1,2; \zeta_i = O(\varepsilon), i = 1, \cdots, 4; \beta_{2j} = O(\varepsilon), j = 1,2. \tag{2}$$

The authors have investigated the stability of the solutions by using the method of computational and numerical analysis. The solution was obtained by using the integration of equations of motions performing a Fourth Order Runge-Kutta Method. The behavior of a nonlinear main system with nonlinear secondary system also have been investigated to many cases of resonances.

On the other hand, under what conditions the system will appear oscillation is also important. Therefore, in this paper we discuss the oscillatory behavior of the solution for the model (1). Our result indicates that if the autonomous system associated with (1) has an oscillatory solution, then there exists an oscillatory solution of system (1) since $F_1 \cos(\Omega_1 t) + y_2 F_2 \cos(\Omega_2 t)$ is an external periodic force.

2 Preliminaries

For convenience, system (1) can be written as an equivalent four dimensional first order system:

$$\begin{cases} x_1' = x_2, \\ x_2' = -\omega_1^2 x_1 - \alpha_{12} x_1^2 - \alpha_{13} x_1^3 - \zeta_1 x_2 - \zeta_2 x_1^2 x_2 + \alpha_{21} x_3 - \zeta_3 (x_2(t-\tau) - x_4(t-\tau)) \\ \qquad -\alpha_{22}(x_1 - x_3)^2 - \alpha_{23}(x_1 - x_3)^3 + F_1 \cos(\Omega_1 t) + x_3 F_2 \cos(\Omega_2 t), \\ x_3' = x_4, \\ x_4' = -\omega_2^2 (x_3 - x_1) - \beta_{22}(x_3 - x_1)^2 - \beta_{23}(x_3 - x_1)^3 - \zeta_4 (x_4(t-\tau) - x_2(t-\tau)). \end{cases} \tag{3}$$

Corresponding to system (3), we have the following autonomous system:

$$\begin{cases} x_1' = x_2, \\ x_2' = -\omega_1^2 x_1 - \alpha_{12} x_1^2 - \alpha_{13} x_1^3 - \zeta_1 x_2 - \zeta_2 x_1^2 x_2 + \alpha_{21} x_3 - \zeta_3 (x_2(t-\tau) - x_4(t-\tau)) \\ \qquad -\alpha_{22}(x_1 - x_3)^2 - \alpha_{23}(x_1 - x_3)^3, \\ x_3' = x_4, \\ x_4' = -\omega_2^2 (x_3 - x_1) - \beta_{22}(x_3 - x_1)^2 - \beta_{23}(x_3 - x_1)^3 - \zeta_4 (x_4(t-\tau) - x_2(t-\tau)). \end{cases} \tag{4}$$

The system (4) can be expressed in the following matrix form:

$$x'(t) = Ax(t) + Bx(t-\tau) + f(x(t)) \tag{5}$$

where $x = (x_1, x_2, x_3, x_4)^T$, $x(t-\tau) = (x_1(t-\tau), x_2(t-\tau), x_3(t-\tau), x_4(t-\tau))^T$, A and B both are 4 by 4 matrices, and $f(x(t))$ is a 4 by 1 vector:

$$A = (a_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & -\zeta_1 & \alpha_{21} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\omega_2^2 & \omega_2^2 & 0 \end{pmatrix},$$

$$B = (b_{ij})_{4\times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\zeta_3 & 0 & \zeta_3 \\ 0 & 0 & 0 & 0 \\ 0 & \zeta_4 & 0 & -\zeta_4 \end{pmatrix},$$

$$f(x) = \begin{pmatrix} 0 \\ -\alpha_{12}x_1^2 - \alpha_{13}x_1^3 - \zeta_2 x_1^2 x_2 - \alpha_{22}(x_1 - x_3)^2 - \alpha_{23}(x_1 - x_3)^3 \\ 0 \\ -\beta_{22}(x_3 - x_1)^2 - \beta_{23}(x_3 - x_1)^3 \end{pmatrix}.$$

The linearized system of (5) is

$$x'(t) = Ax(t) + Bx(t - \tau) \tag{6}$$

Lemma 1 If $\beta_{22}^2 - 4\alpha_{13}\beta_{23} < 0, \alpha_{12}^2 - 4\alpha_{13}(\omega_1^2 - \alpha_{21}) < 0$, then there exists a unique equilibrium point for system (4) (or (5)).

Proof An equilibrium point $x^* = [x_1^*, x_2^*, x_3^*, x_4^*]^T$ of system (4) is a constant solution of the following algebraic equation

$$\begin{cases} x_2^* = 0, \\ -\omega_1^2 x_1^* - \alpha_{12}x_1^{*2} - \alpha_{13}x_1^{*3} - \zeta_1 x_2^* - \zeta_2 (x_1^*)^2 x_2^* + \alpha_{21}x_3^* \\ \qquad -\zeta_3(x_2^* - x_4^*) - \alpha_{22}(x_1^* - x_3^*)^2 - \alpha_{23}(x_1^* - x_3^*)^3 = 0, \\ x_4^* = 0, \\ -\omega_2^2(x_3^* - x_1^*) - \beta_{22}(x_3^* - x_1^*)^2 - \beta_{23}(x_3^* - x_1^*)^3 - \zeta_4(x_4^* - x_2^*) = 0. \end{cases} \tag{7}$$

Noting that $x_2^* = 0, x_4^* = 0$, so system (7) changes to the following:

$$\begin{cases} \omega_1^2 x_1^* + \alpha_{12}x_1^{*2} + \alpha_{13}x_1^{*3} - \alpha_{21}x_3^* + \alpha_{22}(x_1^* - x_3^*)^2 + \alpha_{23}(x_1^* - x_3^*)^3 = 0, \\ \omega_2^2(x_3^* - x_1^*) + \beta_{22}(x_3^* - x_1^*)^2 + \beta_{23}(x_3^* - x_1^*)^3 = 0. \end{cases} \tag{8}$$

We shall prove that $x_1^* = 0, x_3^* = 0$. Indeed, from the second equation of (8), we have $x_3^* - x_1^* = 0$, or $\beta_{23}(x_3^* - x_1^*)^2 + \beta_{22}(x_3^* - x_1^*) + \omega_2^2 = 0$. Condition $\beta_{22}^2 - 4\alpha_{13}\beta_{23} < 0$ implies that there are no real roots of equation $\beta_{23}(x_3^* - x_1^*)^2 + \beta_{22}(x_3^* - x_1^*) + \omega_2^2 = 0$. Thus, we only have $x_3^* - x_1^* = 0$. From $x_3^* - x_1^* = 0$ we obtain $x_1^*(\alpha_{13}x_1^{*2} + \alpha_{12}x_1^* - \alpha_{21} + \omega_1^2) = 0$. Condition $\alpha_{12}^2 - 4\alpha_{13}(\omega_1^2 - \alpha_{21}) < 0$ implies that there are no real roots of equation $\alpha_{13}x_1^{*2} + \alpha_{12}x_1^* + \omega_1^2 - \alpha_{21} = 0$. Therefore, we have $x_1^* = 0, x_3^* = 0$. system (4) only have a zero equilibrium point.

Lemma 2 If the trivial solution of system (6) is unstable, then the trivial solution of (5) is unstable.

Proof Obviously, system (5) and (6) both have trivial solution. $f(x)$ is a higher order infinitesimal when $x \to 0$. Therefore, the trivial solution of system (6) is unstable, then the trivial solution of system (5) is unstable.

3 Oscillatory Behavior of the Solutions

Theorem 1 Assume that all solutions of system (3) are bounded. If zero is the unique equilibrium point of system (6) for selecting parameter values. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and $\beta_1, \beta_2, \beta_3, \beta_4$ be characteristic values of matrix A and B, respectively. If there exists some positive α_k, or $Re(\alpha_k) > 0$, then the unique equilibrium point of system (6) is unstable. System (3) generates an oscillatory solution.

Proof Since α_i and β_i $(i = 1, 2, 3, 4)$ are characteristic values of matrix A and B, respectively, then the characteristic equation corresponding to system (6) is the following:

$$\Pi_{i=1}^4(\lambda - \alpha_i - \beta_i e^{-\lambda\tau}) = 0 \tag{9}$$

So, we are led to an investigation of the nature of the roots for some $k, k \in \{1, 2, 3, 4\}$

$$\lambda - \alpha_k - \beta_k e^{-\lambda\tau} = 0 \tag{10}$$

Noting that there exists a zero characteristic value of system B. Without loss of generality, we assume that $\alpha_k > 0$, or $Re(\alpha_k) > 0$, $\beta_k = 0$. Then (10) changes to

$$\lambda - \alpha_k = 0 \tag{11}$$

Since $\alpha_k > 0$, or $Re(\alpha_k) > 0$, this means that there is a positive (or a positive real part) characteristic value of system (6). Therefore, the trivial solution of system (6) is unstable. According to Lemma 2, the trivial solution of (5) is unstable. The boundedness of the solutions of system (5) and the instability of unique equilibrium point will force system (5) to generate an oscillatory solution. Since $F_1 \cos(\Omega_1 t) + y_2 F_2 \cos(\Omega_2 t)$ is a periodic external force, implying that system (3) has an oscillatory solution.

Theorem 2 Let $k = \max\{|\zeta_3|, |\zeta_4|\}, \mu(A) = \max_{1\le j\le 4}[a_{jj} + \sum_{i=1, i\neq j}^{4} |a_{ij}|]$ [21]. Assume that system (3) has a unique equilibrium point and all solutions of system (3) are bounded. If the following inequality holds:

$$\mu(A) + k > 0 \tag{12}$$

then system (3) has an oscillatory solution.

Proof Let $y(t) = \sum_{i=1}^{4} |x_i(t)|$, from (6) we have

$$y'(t) \le \mu(A)y(t) + ky(t-\tau) \tag{13}$$

Consider the scalar differential equation

$$z'(t) = \mu(A)z(t) + kz(t-\tau) \tag{14}$$

According to the comparison theorem of differential equation, we have $y(t) \le z(t)$. For equation (14), the characteristic equation associated with (14) is given by

$$\lambda = \mu(A) + ke^{-\lambda\tau} \tag{15}$$

We claim that there exists a positive characteristic root of equation (15). Indeed, let $g(\lambda) = \lambda - \mu(A) - ke^{-\lambda\tau}$. Then $g(\lambda)$ is a continuous function of λ. From condition (12), we have $g(0) = -\mu(A) - k < 0$. On the other hand, $\lim_{\lambda\to+\infty} e^{-\lambda\tau} \to 0$. Thus, there exists a suitably large positive λ, say λ_1 such that $g(\lambda_1) = \lambda_1 - \mu(A) - ke^{-\lambda_1\tau} > 0$. According to the Intermediate Value Theorem, there exists a λ^*, where $\lambda^* \in (0, \lambda_1)$ such that $g(\lambda^*) = 0$. In other words, λ^* is a positive characteristic root of equation (15), implying that the trivial solution of equation (14) is unstable. Since $y(t) \le z(t)$, this means that the trivial solution of equation (13) is unstable. It suggested that system (3) has an oscillatory solution.

Theorem 3 Assume that system (3) has a unique equilibrium point and all solutions of system (3) are bounded. If the following inequality holds for selecting time delay τ:

$$ke\tau > e^{|\mu(A)|\tau} \tag{16}$$

then system (3) has an oscillatory solution.

Proof We shall prove that the trivial solution of equation (14) is unstable. Suppose this is not the case, then the characteristic equation (15) will have a real nonpositive root, say $\lambda_0 < 0$ such that

$$\lambda_0 = \mu(A) + ke^{-\lambda_0\tau} \tag{17}$$

Thus,

$$|\lambda_0| \geq ke^{-\lambda_0\tau} - |\mu(A)| = ke^{|\lambda_0|\tau} - |\mu(A)| \tag{18}$$

or

$$|\lambda_0| + |\mu(A)| \geq ke^{|\lambda_0|\tau} = ke^{(|\lambda_0|+|\mu(A)|)\tau}e^{-|\mu(A)|\tau} \tag{19}$$

Noting that $e^{(|\lambda_0|+|\mu(A)|)\tau} \geq e(|\lambda_0| + |\mu(A)|)\tau$, thus

$$|\lambda_0| + |\mu(A)| \geq ke(|\lambda_0| + |\mu(A)|)\tau e^{-|\mu(A)|\tau} \tag{20}$$

So we have

$$1 \geq (ke\tau)e^{-|\mu(A)|\tau} \tag{21}$$

The inequality (21) contradicts (16). Therefore, the trivial solution of equation (14) is unstable. Implying that system (3) has an oscillatory solution.

4 Simulation Results

The simulation is based on the equivalent system (3) of (1), first the parameters are selected as follows: $\alpha_{12} = 0.03, \alpha_{13} = 0.04, \alpha_{21} = 0.87, \alpha_{22} = 0.35, \alpha_{23} = 0.025; \beta_{22} = 0.98, \beta_{23} = 1.32; \zeta_1 = 0.05, \zeta_2 = 0.08, \zeta_3 = 0.04, \zeta_4 = 2; \omega_1 = 2.25, \omega_2 = 6.35; \Omega_1 = 0.45, \Omega_2 = 0.55; F_1 = 0.6, F_2 = 0.3$. The time delay $\tau = 0.185$. Then the characteristic values of $A = [0\ \ 1\ \ 0\ \ 0; -2.25^2\ \ -0.05\ \ 0.87\ \ 0; 0\ \ 0\ \ 0\ \ 1; 0\ \ -6.35^2\ \ 0]$ are $5.9319, 0.3650 \pm 2.2340i, -6.7119$. Since there is a positive characteristic value (5.9319) of matrix A, the condition of Theorem 1 are satisfied. One can check that the conditions of Theorem 2 are also satisfied. There exists an oscillatory solution for system (3). However, the oscillatory amplitude of the solution is small (see Fig.1). Then we change $\omega_1 = 2.15, \omega_2 = 2.35, \Omega_1 = 1.85, \Omega_2 = 1.55$, the other parameters are kept as before, both oscillatory amplitude and frequency of the solution are changed (see Fig.2), implying that the values of ω_1, ω_2, Ω_1, and Ω_2 affect the oscillatory amplitude and frequency of the solution greatly. In order to see the effect of time delay, we use the parameters in figure 1 and change the time delay τ from 0.185 to 0.225 and 0.285, respectively. One can see that both the oscillatory amplitude and frequency are changed, implying that time delay affects amplitude and frequency of the oscillation (see Fig.3). We pointed out that the condition of Lemma 1 is not satisfied based on selecting parameter values. This means that Lemma 1 only is a sufficient condition.

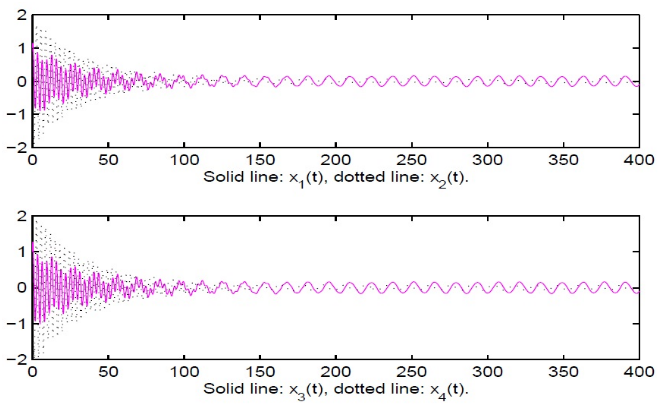

Fig.1 Oscillatory behavior of the solutions for system (3), delay: 0.185.
w_1=2.25, w_2=6.35, Omega$_1$=0.45, Omega$_2$=0.55.

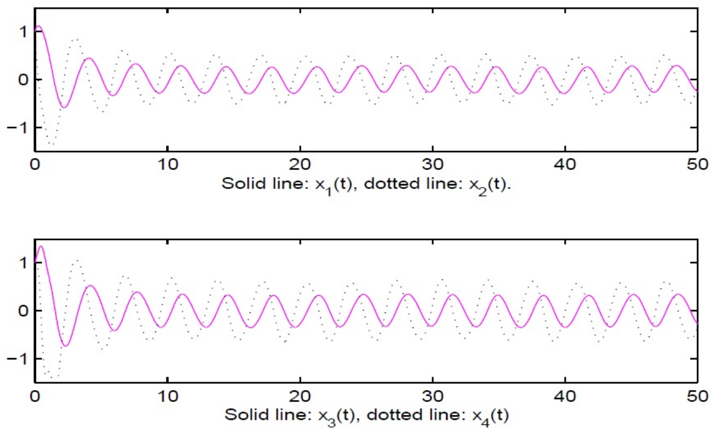

Fig. 2 Oscillatory behavior of the solutions for system (3), delay: 0.185. w_1=2.15, w_2=2.35, Omega$_1$=1.85, Omega$_2$=1.55.

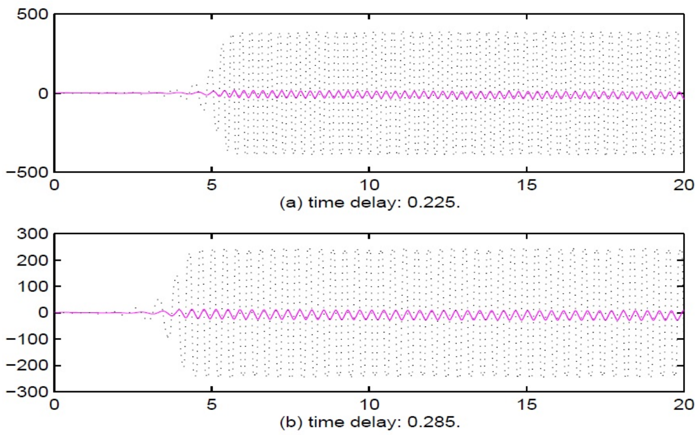

Fig.3 The effect of time delays, solid line: $x_3(t)$, dotted line: $x_4(t)$.

5 Conclusion

In this paper, we have discussed the oscillatory behavior of the solutions for a class of nonlinear mechanical system with delay. Based on mathematical analysis method, we provided some sufficient conditions to guarantee the oscillation of the solutions. Some simulations are provided to indicate the effectness of the criterion.

Competing Interests

Author has declared that no competing interests exist.

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