



A Practical Robust Direct Model Anti-windup Design

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper considers the synthesis of robust anti-windup compensators using direct models for stable multivariable linear time invariant uncertain plants. An internal model control antiwindup compensator as a special case of the direct model, preserves the robust stability of the unconstrained linear closed-loop system. The proposed direct model antiwindup aims to improve the performance of the constrained closed-loop system while guaranteeing robust stability. Robust stability and performance are achieved by shaping two closed-loop transfer functions based on a decoupled representation of the constrained closed-loop. Trade offs between robust stability and robust performance are provided by adjusting performance and stability weights. The direct model synthesis is cast into a \mathcal{H}_∞ optimization or a μ synthesis problem, where the anti-windup objectives are robust stability and rapid recovery of the linear response. Two simulation examples, one single input single output(SISO), one multi-input multi-output(MIMO), demonstrate the design process and effectiveness of the proposed scheme.

Keywords: Direct model; anti-windup; MIMO; robust control design.

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1. INTRODUCTION

In applications, all actuators have physical output limits. Actuator saturation leads to performance degradation and, sometimes, instability in the feedback systems. A large amount of research dealing with saturation effects on control systems' stability and performance, has been continuously conducted since 1950's(see [1],[2] and references therein). The linear antiwindup (AW) augmentation is a popular approach: a linear controller is first designed based on the linear control theory ignoring the fact that the actuator can saturate, then add the antiwindup compensation to counteract saturation in case it occurs. The philosophy of designing the AW compensator is twofold [3],[4]: (1) to preserve the stability and performance properties of the unconstrained linear system in the absence of saturation and; (2) to swiftly recover the linear response when saturation occurs while maintaining the stability of the closed-loop system.

IMC [5] [6] was not intended for anti-windup, however, the IMC guarantees the closed-loop stability under input saturation provided that both the plant and the controller are stable. [7],[8] showed that the IMC anti-windup scheme is optimal in the sense of the retention of the linear system's stability with respect to additive plant uncertainty. But many examples have shown IMC AW to have poor performance [9], [7], [10] when the plant has slow dynamics or non-minimum phase zeros. Direct model introduced in [11] was attributed to the unpublished work by Irving. In the special case when the direct model is the plant model, the direct model AW becomes the IMC AW[12]. So it is natural to seek to improve the AW performance by synthesizing a direct model which may not be the plant model. The two AW design objectives mentioned earlier are used as the design guidelines here. The technique of decomposing the constrained system using the loop transmission around the saturating actuator $L_n(s)$ [13] [8] is applied to reveal two closed-loop transfer functions which essentially determine nonlinear stability and performance of the constrained system respectively. This decomposition naturally leads to the synthesis of the direct model anti-windup compensator. A \mathcal{H}_∞

optimization or μ synthesis problem is formulated to obtain a direct model, which is solved easily by widely available control softwares. Robust stability and robust performance are traded off by introducing weighting functions into the optimization. By increasing the stability weight, the direct model approaches the IMC AW, namely the plant model, thus recovers the robust stability of the linear design. In the opposite, increasing the performance weight improves the AW performance. The design transparency offered by this design process is desirable in practice.

Another notable aspect of the proposed scheme is that it considers plant uncertainty. Including plant uncertainty in AW design is not a trivial task, many AW approaches do not work if there are uncertainties in the loop. However, without the robustness consideration, we are exposed to failure in real life applications. Robustness AW research remains an important area to be further developed; some works in the area include [15],[8],[7],[14].

By both considering plant uncertainty and offering stability and performance tradeoffs through the common framework of \mathcal{H}_∞/μ synthesis, with commercial software packages readily available and numerical simulation made easier, the proposed approach is specially suited to practical engineering applications.

This paper is organized as follows. In Section 2, we introduce the direct model AW problem. In section 3, we address the AW synthesis problem. Two numerical examples are given in Section 4. Finally, conclusions are drawn in Section 5.

2. ANTIWINDUP PROBLEM

2.1 Constrained System Description

Consider the direct model AW in Fig. 1 where $P(s)$ represents the true plant model with $P_0(s)$ the linear model of the nominal plant, $G(s) = [G_r(s) G_c(s)]$ represents the 2DOF linear controller. We assume that the plant is Hurwitz to have global AW properties.

When $P_m = P_0$, the AW in Fig. 1 becomes the well known IMC AW. The stability properties of IMC AW are its key advantage. When the plant has no uncertainty, the IMC AW system is always stable as long as the linear feedback controller G_c and the plant P are stable. When the plant has additive uncertainty, it was shown in [7] that the IMC AW closed-loop is still robustly stable for the same uncertainty class provided that the linear system is robustly stable. In Fig. 1, define the loop transmission around the saturation block $L_n(s)$ as $-y_c(s) = L_n(s)u(s)$, this transfer function is critical for AW compensation [13].

$$L_n = (I - G_c P_m)^{-1} G_c (P_m - P). \quad (0.1)$$

$$P(I + L_n)^{-1} = P(I - G_c P)^{-1} (I - G_c P_m). \quad (0.2)$$

$$L_n(I + L_n)^{-1} = (I - G_c P)^{-1} G_c (P_m - P). \quad (0.3)$$

A system decomposition technique using $L_n(s)$ was proposed in [13], where it was emphasized that AW is achieved by properly modifying this loop transmission through introducing AW

compensation. Manipulating L_n to achieve saturation compensation was first performed by Horowitz [16] and was adopted afterwards [17],[15],[18]. Applying this technique, we have the following decomposed AW system in Fig. 2. Note that in Fig. 2, the saturation block is replaced by the deadzone nonlinearity. It is worth pointing out that this decomposition holds when the plant is uncertain, which is not the case for the $M(s)$ conditioning in [9]. In Fig. 2, the AW constrained system is decomposed into three components: the linear loop, the nonlinear loop and the disturbance filter. The linear loop gives the desired linear response, the nonlinear loop containing L_n affects the nonlinear stability, and the disturbance filter also containing L_n determines the nonlinear performance. It is evident that L_n , as an additional design degree of freedom, can be designed independently and properly for saturation. This is achieved here by influencing L_n using a direct model P_m .

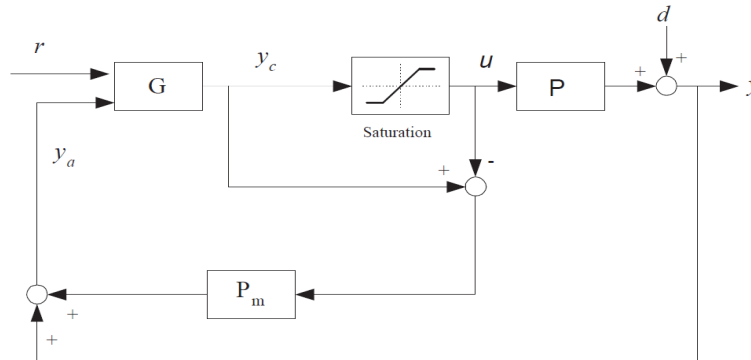


Fig. 1. Direct model AW Structure

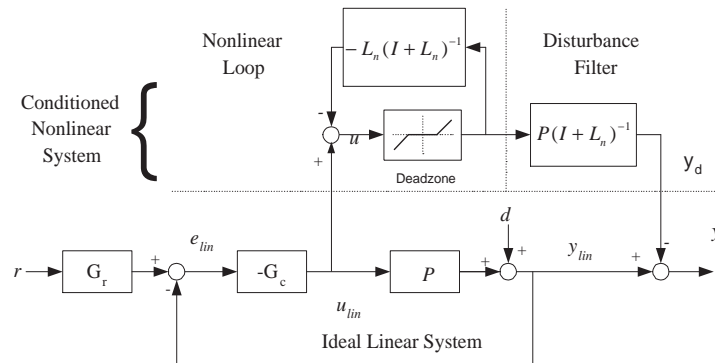


Fig. 2. Decoupled representation of the AW system

3. ANTI-WINDUP SYNTHESIS

According to the decomposition, the nonlinear stability is guaranteed if the \mathcal{H}_∞ norm of $L_n(I + L_n)^{-1}$ is less than 1 based on the small gain theorem, and the nonlinear response is close to the linear one if the \mathcal{H}_∞ norm of $P(I + L_n)^{-1}$ is small. So we have an optimization problem for AW synthesis for the following system

$$e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} W_p P(I + L_n)^{-1} \\ W_s L_n(I + L_n)^{-1} \end{pmatrix} w \quad (0.4)$$

where the performance weight W_p and the stability weight W_s are introduced to balance the robust performance and the robust stability, and we would like to minimize the \mathcal{H}_∞ norm from the input w to error e . This problem can be arranged into the standard \mathcal{LFT} form, Fig. 3, where

$$e_1 = W_p P(I - G_c P)^{-1} w + W_p P G_c (I - P G_c)^{-1} y_m \quad (0.5)$$

$$e_2 = W_s G_c P(I - G_c P)^{-1} w + W_s P G_c (I - P G_c)^{-1} y_m \quad (0.6)$$

$$u_m = -w \quad (0.7)$$

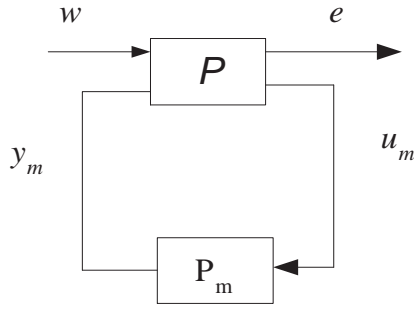


Fig. 3. Standard design form

Now, this optimization problem can be solved either as a standard \mathcal{H}_∞ optimization problem using the nominal plant model or a μ synthesis problem including plant uncertainty. After the direct model is obtained, the \mathcal{H}_∞ norms of $L_n(I + L_n)^{-1}$ and $P(I + L_n)^{-1}$ are computed to check the stability and performance. If the specified stability or performance level is not achieved with the current direct model, adjust weights and repeat the design process. The weights are tuned by the designer to achieve the desired balance between stability and performance in the design process. As guaranteed by the IMC AW, when stability is weighed over performance, more robust stability of the linear system is recovered in the compensated constrained system. Both approaches were demonstrated in the second example.

When the plant uncertainty or nonlinearity is not full block, instead it has a structure, i.e. diagonal,

the μ synthesis has a higher potential to reduce the conservatism as compared to the standard LMI-based linear graph separation techniques (the bounded real lemma, the passivity, the small gain theorem, etc), which typically convexifies the nonconvex constraints by accounting for full block uncertainties/nonlinearities in the LFT factorization. In the case of structured uncertainties, μ synthesis can be performed to solve the proposed optimization problem to improve performance compared to a \mathcal{H}_∞ optimization. μ synthesis, i.e. \mathcal{DK} iteration, can be conducted more effectively due to recent robust control softwares.

The direct model resulting from the design process could be of high order. In implementation, model reduction techniques can be applied to obtain a reduced order direct model, and the performance will not be compromised. This is demonstrated in the second example.

4. SIMULATION STUDY

4.1 SISO Example

This example is from [10] and was also used in [12]. It was intended to show the limitations of existing anti-windup approaches. The plant model comprises two second order Butterworth filters in series, where

$$P = 0.2 \frac{s^2 + 2\zeta_1 \omega_1 s + \omega_1^2}{s^2 + 2\zeta_1 \omega_2 s + \omega_2^2} \frac{s^2 + 2\zeta_2 \omega_1 s + \omega_1^2}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} \quad (0.8)$$

with $\omega_1 = 0.2115$, $\omega_2 = 0.0473$, $\zeta_1 = 0.3827$, and $\zeta_2 = 0.9239$. and the controller is an integrator,

$$G_c = -5/s. \quad (0.9)$$

Control is limited between ± 1 . Without AW compensation, the constrained response is a limit cycle of large amplitude.

We chose the direct model as

$$P_0 = \frac{4s + 1}{20s + 1}. \quad (0.10)$$

The linear response and the direct model AW response under a unit step is shown in Fig. 4. The proposed approach gave very good response, comparable to the linear one. The direct model gave better responses than IMC, the CAW in [10] and the approach in [12]. This example shows a low order direct model could be effective based on simple design concepts.

4.2 MIMO Plant with Uncertainty

This example is from [6]. We added uncertainty to the plant in order to apply the robust design approach. Nominal plant is

$$P_0 = \frac{10}{100s + 1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \quad (0.11)$$

the feedback controller is

$$-G_c = \begin{bmatrix} \frac{2s+0.02}{1.5s+0.015} & \frac{2.5s+0.025}{2s+0.02} \\ \frac{s}{s} & \frac{s}{s} \end{bmatrix} \quad (0.12)$$

We added input multiplicative to the plant such that

$$P = P_0(I + W\Delta), \quad \|\Delta\|_\infty < 1 \quad (0.13)$$

and W gives the size of the uncertainty, which indicates 20% modeling error at low frequencies and 100% modeling error above 3rad/sec, see Fig. 5.

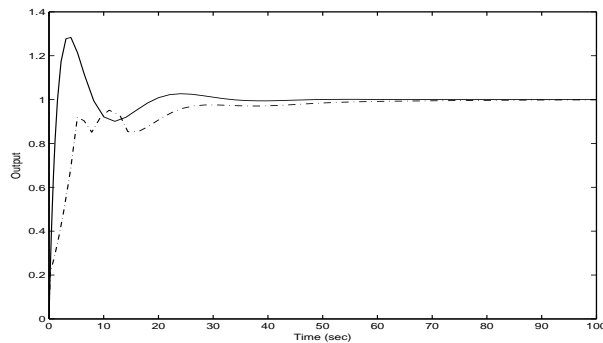


Fig. 4. Step responses: linear response, solid line; direct model AW, dash-dotted line

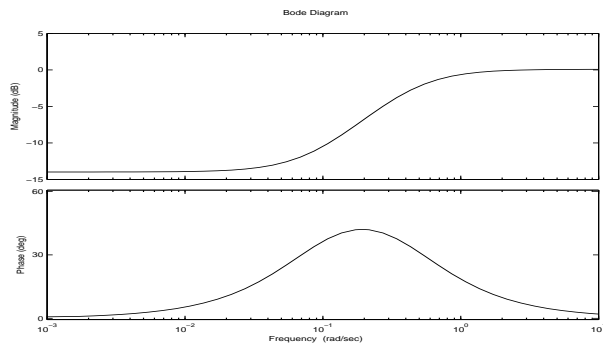


Fig. 5. Step responses: linear response, solid line; direct model AW, dash-dotted line

For robust direct model design, we choose weights as

$$W_p = I_{2 \times 2} \quad (0.14)$$

$$W_s = 12 \times I_{2 \times 2}. \quad (0.15)$$

\mathcal{H}_∞ optimization and μ synthesis were executed to get two direct model AW compensators, using Matlab robust control toolbox. The performance of the AW compensators in the worst case are compared in Table 1, where all bounds are upper bounds except the bound of $P(I + L_n)^{-1}$ of IMC is lower bound, because Matlab returned infinity for the upper bound. Based on the table, both direct models guarantee the robust stability for the specified class of uncertainties, because of $\|L_n(I + L_n)^{-1}\|_\infty < 1$, and significantly improve the robust performance compared to IMC AW because of a much smaller $\|P(I + L_n)^{-1}\|_\infty$.

In the simulation, step inputs [0.63 0.79] are applied, and both control inputs are constrained between ± 1 . Fig. 6 shows the linear and constrained response for the nominal plant model. Responses with AW action for the nominal plant are shown in Fig. 7, where

both direct models improved the response compared to IMC AW and their responses are indistinguishable.

Next, we compare the robust performance of three AW compensators. In Fig. 8, the uncertainty that gave the worst gain $\|P(I + L_n)^{-1}\|_\infty = 3.5633$ for the direct model designed using \mathcal{H}_∞ optimization was used to compare the robust performance of the \mathcal{H}_∞ direct model AW and the IMC AW $G_m = P_0$. In Fig. 9, the uncertainty that gave the worst gain $\|P(I + L_n)^{-1}\|_\infty = 3.7079$ for the direct model designed using μ synthesis was used to compare the robust performance of the μ direct model AW and the IMC AW using the nominal plant model. In both cases, the direct model AW gave much improved response over the IMC AW. Fig. 10 and Fig. 11 compare the singular value plot of the closed-loop transfer functions $L_n(I + L_n)^{-1}$ and $P(I + L_n)^{-1}$ of three AW compensators under their worst case uncertainties, respectively. It can be seen that IMC AW is too conservative, thus giving poor performance, whilst the direct models not only meet the stability requirement but also give better performance.

Table 1. AW Performance

Worst case bound	\mathcal{H}_∞	μ	IMC
$L_n(I + L_n)^{-1}$	0.9978	0.9893	0.25
$P(I + L_n)^{-1}$	3.5633	3.7079	81.3015

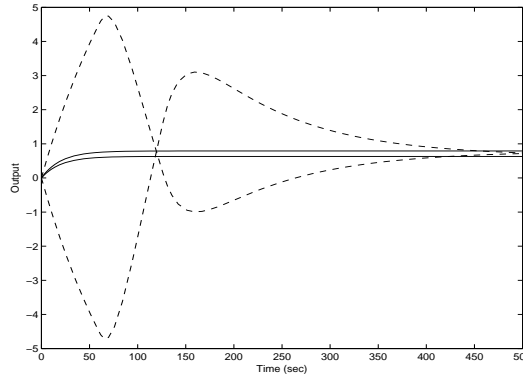


Fig. 6. Nominal responses:linear, solid;constrained, dashed

In Fig. 12, singular values of the the three AW compensators are compared. The direct model will approach the nominal plant which is the IMC AW, if W_s becomes larger and larger. The robust stability of the linear system will be recovered with the loss of the performance. The order of the \mathcal{H}_∞ direct model is 6, and the order of the μ direct

model is 14. Time responses shown earlier were obtained using full order direct models. Applying model reduction, the order of both direct models could be reduced to 3 and the system responses with reduced order direct models were very close to those using the full order direct models.

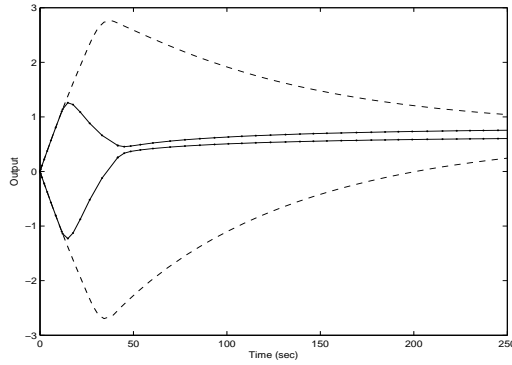


Fig. 7. Nominal AW responses: μ , solid; \mathcal{H}_∞ , point; IMC, dashed

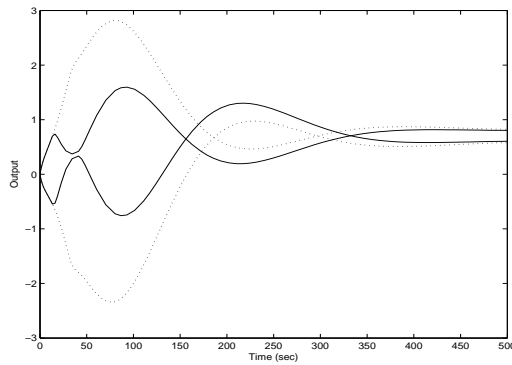


Fig. 8. Robust AW responses: \mathcal{H}_∞ , solid; IMC, dotted

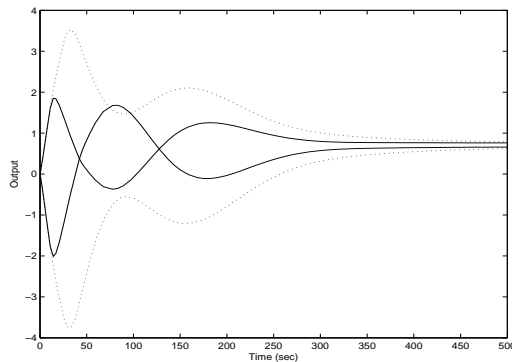


Fig. 9. Robust AW responses: μ , solid; IMC, dotted

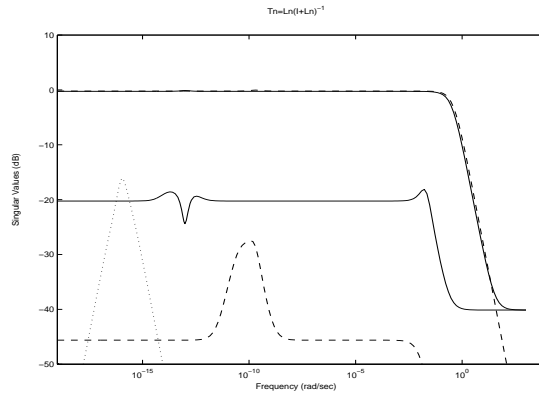


Fig. 10. Robust stability: μ , solid; \mathcal{H}_∞ , dashed;IMC, dotted

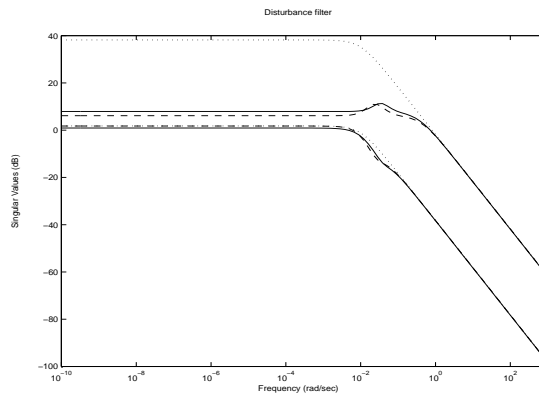


Fig. 11. Robust stability: μ , solid; \mathcal{H}_∞ , dashed;IMC, dotted

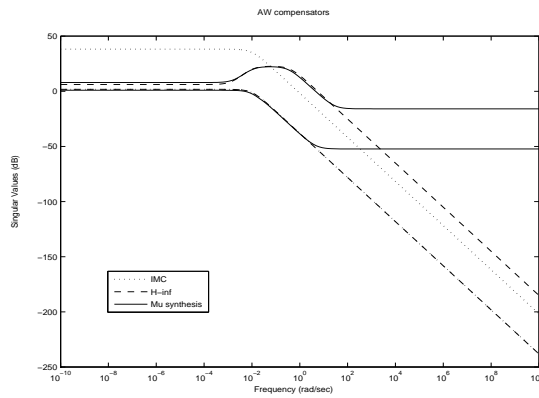


Fig. 12. AW compensators: μ , solid; \mathcal{H}_∞ , dashed;IMC, dotted

5. CONCLUSIONS

Synthesis of direct models for anti-windup is proposed. The resulting direct model improves the anti-windup performance with a specified robust stability level. Plant uncertainty is accounted for and the stability and performance are captured in the design by shaping two closed-loop transfer functions based on decomposition. μ synthesis is utilized to reduce the conservatism for structured uncertainties. The trade-off between stability and performance is achieved through adjusting weights. The proposed approach is practical and suited for engineering applications. Two examples demonstrated the design concepts and the improved anti-windup performance under plant uncertainty compared to other AW results in the literature.

COMPETING INTERESTS

The author declares that no competing interests exist.

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