



Comparison of Suction/Injection Effect on Flow, Heat and Mass Transfer in Porous Media Using a Combined Similarity-Numerical Solution

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Authors' contributions

This work was carried out in collaboration among all authors. Author FA designed the study; authors AAA performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors BOP and AAA managed the analyses of the study and the literature searches. All authors read and approved the final manuscript

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ABSTRACT

This investigation deals with a comparison of suction/injection effect on flow, heat and mass transfer in porous media using a combined similarity-numerical solution. With this method of transformation, the governing transport PDEs are transformed into ODE and then solved numerically. The study reveals that suction/injection effect is more pronounced on the velocity distribution of a creeping flow (Darcy flow). The Darcy-Forchheimer flow has the steepest velocity curves due to non-linearity and has higher skin friction, heat and mass transfer rate when compared to the other porous media investigated.

Keywords: Suction/Injection; porous media; heat and mass transfer; Darcy-Forchheimer; similarity-numerical solution.

1. INTRODUCTION

The effect of suction/injection on flow, heat and mass transfer in porous media has been an

active field of research in engineering as it plays a crucial role in diverse applications, such as boundary layer control, catalytic reactors and transpiration cooling. A considerable number of

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researchers have reported their findings on flow, heat and mass transfer in Darcian, non-Darcian and Darcy-Brinkman-Forchheimer porous media. Some of these researchers include Kaviany [1] who used the Darcy-Brinkman model to study the effects of boundary and inertia forces on the forced convection over a fixed impermeable heated plate embedded in a porous medium Kumari M., Pop I., and Nath G [2] also works on forced convection heat transfer where the investigated the non-Darcian effect on the forced convection heat transfer over a flat plate in a highly porous medium. Considerable attention has been paid to theoretical and numerical study of flow and heat transfer in porous media in the recent past as it plays a crucial role in diverse applications, such as advance nuclear reactor cooling, reservoir maintenance, extraction of crude oil etc.

Different types of fluids have been investigated in several kinds of porous media. Buongiorno [3], made a survey of convective heat transfer in nanofluid, where he pointed out that several authors have concluded that convective heat transfer of nanofluid is enhanced by the dispersion of nanoparticles but he argued that this effect is very small to explain the abnormal enhancement. He also explained that turbulence and particle rotation are not the cause of heat transfer enhancement in nanofluid and that their effect is too small to explain the effect. He later concluded from his model that the enhanced heat transfer characteristics and the abnormal increase in the thermal conductivities of a nanofluid is as a result of the Brownian diffusion and the thermophoretic diffusion of the nanoparticles in the absence of turbulence. Recently, S. U. Rahman et al. [4] investigated the simultaneous effects of nanoparticles and slip on Jeffrey fluid through tapered artery with mild stenosis. Moreover, A. Zeeshan, A. Majeed and R. Ellahi [5] studied the Effect of magnetic dipole on viscous Ferro-fluid past a stretching surface with thermal radiation. Mohsen Sheikholeslami and Rahmat Ellahi [6] studied electro hydrodynamic nanofluid hydrothermal treatment in an enclosure with sinusoidal upper wall. Sheikholeslami [7], discussed the effect of thermal radiation on nanofluid flow and heat transfer using two phase model. Ellahi, Rahmat; Hassan, Mohsin and Zeeshan, Ahmad [8], studied problems on natural convection Magneto-hydrodynamics, MHD nanofluid by means of single and multi-walled carbon nanotubes suspended in a salt water solution.

With the addition of a heat sink, the work of Ajibade et al. [9] shows that the velocity and temperature profiles have been shown to drop. This is physically correct because when heat is absorbed, the fluid gets denser and the convection current weakens, lowering the fluid temperature and, as a result, lowering the fluid flow. It is also noted that when heat generation increases, both the velocity and temperature profiles rise. Increased heat generation enhances the convection current, which lowers the density of the fluid, raising the temperature and increasing the fluid velocity. In the work of S.S Mustafa and Rasha A. Mohamed [10] the effect of suction/injection was examined. It was detailed that the suction leads to a decrease in surface mass transfer whereas Injection causes an increase in the surface heat transfer.

From the literature survey, various researches has been carried out on the effect of suction/injection on fluid flow, few had barely considered suction/injection effect on heat and mass transfer in porous media. Therefore, our present study aimed to explore the similarity solutions of the transformed nonlinear ODE's for different porous media. In each model, the controlling parameter determines the type of media, that is to say for a Darcy model, the inertial parameter is absent by setting $\kappa_2 = 0$, for a non-Darcy model, the permeability parameter is absent by setting $\kappa_1 = 0$ and for a Darcy-Forchheimer model, both controlling parameters are present (permeability and inertial). It is important to note that Darcy's classical flow model is a fundamental law relating the pressure gradient, viscosity and fluid velocity linearly through porous medium. Any deviation from Darcy's law and assumptions is termed as non-Darcy flow

1.1 Formulation of the Problem

Consider a steady two-dimensional, boundary layer flow of a nanofluid over a porous plate embedded in a porous medium. The x -axis is taken along the surface of the porous plate and the y -axis perpendicular to it. The temperature T_w and concentration C_w on the surface of the plate are kept constant, and assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞ , respectively. The governing equations of the flow field can be written in dimensional form as:

$$\rho_f \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_f \frac{\partial^2 u}{\partial y^2} - \frac{\mu_f}{\kappa} (u - u_\infty) - \frac{F_0'}{x^2} \frac{\rho_f}{(F_0 x)^{\frac{1}{2}}} (u^2 - u_\infty^2) \quad (1)$$

$$(\rho_c)_f \left(u \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} \right) = \kappa_m \frac{\partial^2 T_f}{\partial y^2} + (\rho_c)_p \left[D_B \frac{\partial C_f}{\partial y} \frac{\partial T_f}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T_f}{\partial y} \frac{\partial T_f}{\partial y} \right] \quad (2)$$

$$u \frac{\partial C_f}{\partial x} + v \frac{\partial C_f}{\partial y} = D_B \frac{\partial^2 C_f}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T_f}{\partial y^2} \quad (3)$$

where u and v are the velocity components in the x and y directions respectively, u_∞ is the free stream velocity, μ_f is the viscosity, T_f is the temperature of the nanofluid, C_f is the concentration of the nanofluid, T_w is the temperature along the porous plate, C_w is the concentration along the porous plate, T_∞ and C_∞ are the ambient temperature and concentration respectively, $F = F_0 x$ is the Darcy permeability of the porous medium, F_0 is the inertial parameter, $F' = \frac{F_0'}{x^2}$ is the Forchheimer resistance and F_0' is the Forchheimer constant, D_B is the Brownian motion coefficient, D_T is the thermophoresis coefficient, κ_m is the thermal conductivity, $(\rho_c)_p$ is the heat capacitance of the nanoparticles, $(\rho_c)_f$ is the heat capacitance of the base fluid.

The boundary conditions for velocity, temperature and concentration components are given by:

$$u = 0, v = v_w, T_f = T_w, C_f = C_w \text{ at } y = 0 \quad (4a)$$

$$u \rightarrow u_\infty, T_f \rightarrow T_\infty, C_f \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (4b)$$

The equation of continuity is satisfied automatically with $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, where $\psi(x, y)$ is the stream function.

2. ANALYSIS

The stream function $\psi(x, y)$ is introduced as:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (5)$$

By using Eq. (5), the momentum equation Eq. (1), energy equation Eq. (2) and the concentration equation Eq. (3) takes the following form:

$$\rho_f \left(\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right) = \mu_f \frac{\partial^3 \psi}{\partial y^3} - \frac{\mu_f}{\kappa} \left(\frac{\partial \psi}{\partial y} - u_\infty \right) - \frac{F_0'}{x^2} \frac{\rho_f}{(F_0 x)^2} \left(\frac{\partial^2 \psi}{\partial y^2} - u_\infty^2 \right) \quad (6)$$

$$(\rho_c)_f \left(\frac{\partial \psi}{\partial y} \frac{\partial T_f}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T_f}{\partial y} \right) = \kappa_m \frac{\partial^2 T_f}{\partial y^2} + (\rho_c)_p \left[D_B \frac{\partial C_f}{\partial y} \frac{\partial T_f}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T_f}{\partial y} \frac{\partial T_f}{\partial y} \right] \quad (7)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial C_f}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C_f}{\partial y} = D_B \frac{\partial^2 C_f}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T_f}{\partial y^2} \quad (8)$$

The boundary conditions in Eq. (4a) and (4b) for the velocity components reduce to

$$\frac{\partial \psi}{\partial y} = 0, \frac{\partial \psi}{\partial x} = 0, \text{ at } y = 0; \frac{\partial \psi}{\partial y} \rightarrow U_\infty \text{ as } y \rightarrow \infty \quad (9)$$

We now introduce the dimensionless variables for ψ, T_f and C_f below

$$\psi(x, y) = \sqrt{u_\infty v x} f(\eta), \theta(\eta) = \left(\frac{T_f - T_\infty}{T_w - T_\infty} \right) \text{ and } \phi(\eta) = \left(\frac{C_f - C_\infty}{C_w - C_\infty} \right) \quad (10)$$

where η is the similarity variable and is defined as $\eta = y \sqrt{\frac{u_\infty}{v x}}$.

Using Eq. (10), in equations (6) to (8) we finally obtain the following self-similar equation as:

$$f''' + \frac{1}{2} f f'' - \kappa_1 (f' - 1) - \kappa_2 [f' f' - 1] = 0 \quad (11)$$

$$\frac{\theta''}{Pr} + N_b \phi' \theta' + N_t \theta' \theta' + \frac{1}{2} f \theta' = 0 \quad (12)$$

$$\phi'' + \frac{Nt}{Nb} \theta'' + \frac{Le}{2} f \phi' = 0 \quad (13)$$

The boundary conditions are finally transformed to the following forms:

$$f = \frac{2v_w}{\sqrt{u_\infty v}}, f' = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \quad (14a)$$

$$f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (14b)$$

where $S_n = \frac{2v_0}{\sqrt{u_\infty v}}$ is the suction/injection velocity

$$\kappa_1 = \frac{\mu_f}{\rho F_0 u_\infty}, Da_x = \frac{F_0}{x}, \kappa_2 = \frac{F_0'}{F_0^2}$$

$$Pr = \frac{\nu}{\alpha}, N_b = \frac{\tau}{v} D_B (C_w - C_\infty), N_t = \frac{\tau}{v} \frac{D_T}{T_\infty} (T_w - T_\infty), Le = \frac{v}{D_B}$$

Where k_1 is the permeability parameter, k_2 is the inertia parameter, N_t is the thermophoresis parameter, Pr is the Prandtl number, N_b is the Brownian parameter and Le is the Lewis number.

2.1 Numerical Methods

The coupled sets of non-linear differential equations along with the boundary conditions are

solved numerically using classical fourth order 'Runge-Kutta Method'. In this method, we first convert the governing equations together with the boundary condition into first order system.

$$y'_1 = y_2, y'_2 = y_3, y'_3 = -[0.5y_1y_3 - \kappa_1(y_2 - 1) - \kappa_2(y_2^2 - 1)],$$

$$y'_4 = y_5, y'_5 = -Pr[N_b y_5 y_7 + N_t y_5^2 + 0.5y_1 y_5], \text{ and}$$

$$y'_6 = y_7, y'_7 = -[0.5Le y_1 y_7 + N_{tb} y_5']$$

Where $y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta', y_6 = \phi, y_7 = \phi'$

With the associated boundary conditions

$$f(0) = S_n, f'(0) = 0, \theta(0) = 1 \text{ and } \phi(0) = 1$$

The method employed is the adaptive code which is implemented in MATLAB as an m-file in the form of ODE. It automatically finds the appropriate step size $\Delta\eta$ by comparing the results using the shooting method. It requires size function evaluations per $\Delta\eta$.

3. RESULTS AND DISCUSSION

In other to compare flow characteristics in porous media, numerical computations were carried out. The obtained results are presented graphically to highlight salient features of the flow, the heat and the mass transfer characteristics. Fig. 1 shows Howarth velocity and profile results. A good agreement is found and this supports the validity of our results for other cases. As expected, the velocity profile increases as suction increases and also decreases as injection decreases.

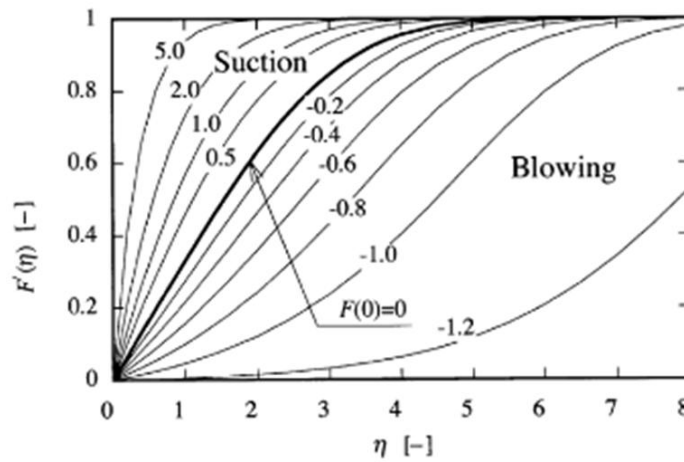


Fig. 1. Variation of velocity distribution $f'(\eta)$ profiles vs η
Source: Howarth [11]

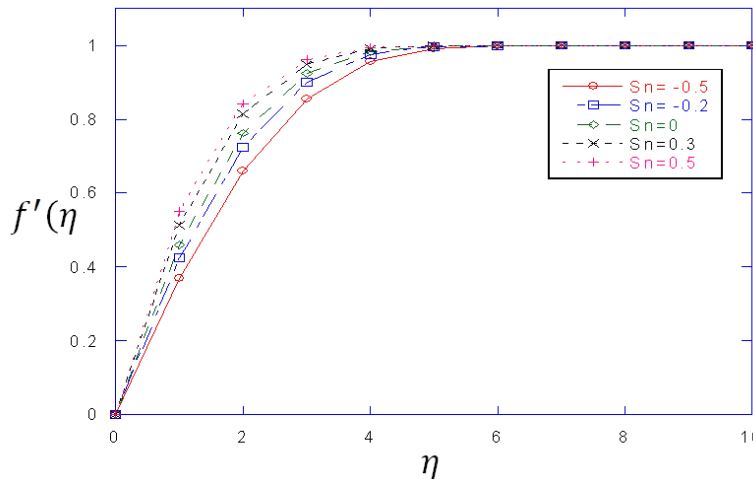


Fig. 2. Velocity distribution vs η (Darcy flow)
 $N_t = N_b = 0.1, Le = 10, Pr = 6, \kappa_1 = 0.2, \kappa_2 = 0$

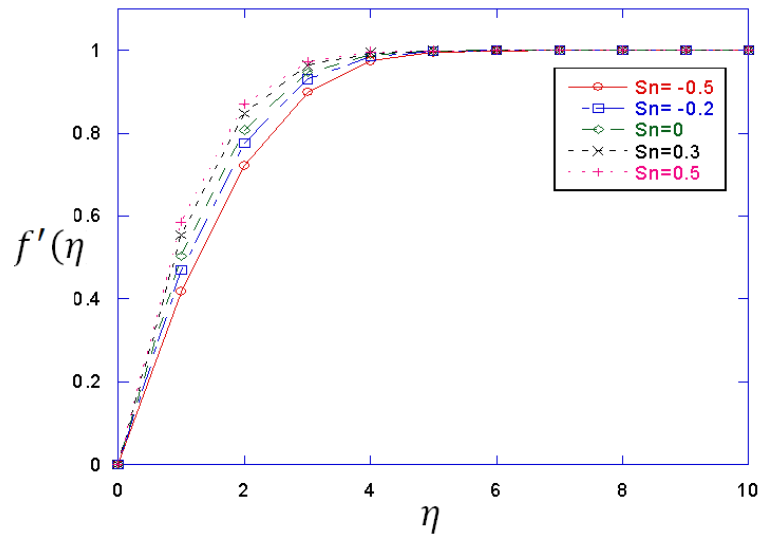


Fig. 3. Velocity distribution vs η (Non – Darcy flow)

$N_t = N_b = 0.1, Le = 10, Pr = 6, \kappa_1 = 0, \kappa_2 = 0.2$

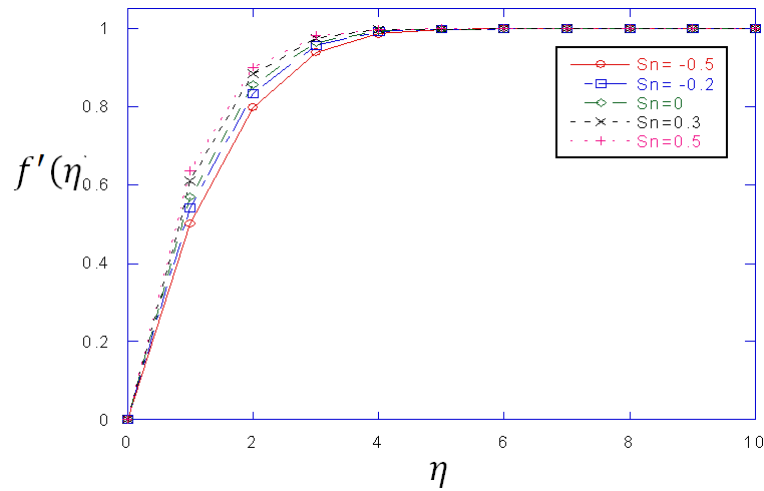


Fig. 4. Velocity distribution vs η (Darcy – Forchheimer flow)

$N_t = N_b = 0.1, Le = 10, Pr = 6, \kappa_1 = \kappa_2 = 0.2$

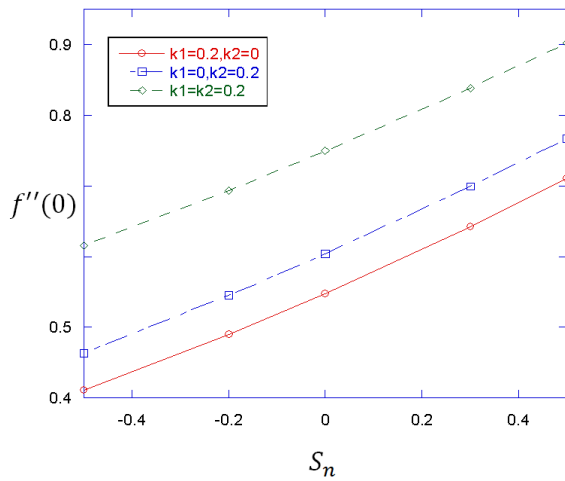


Fig. 5. Skin friction vs S_n
when $N_t = N_b = 0.1, Le = 10, Pr = 6$

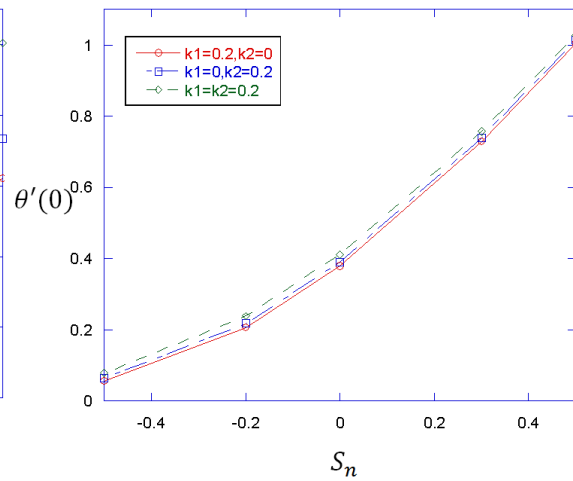


Fig. 6. Heat transfer rate vs S_n
when $N_t = N_b = 0.1, Le = 10, Pr = 6$

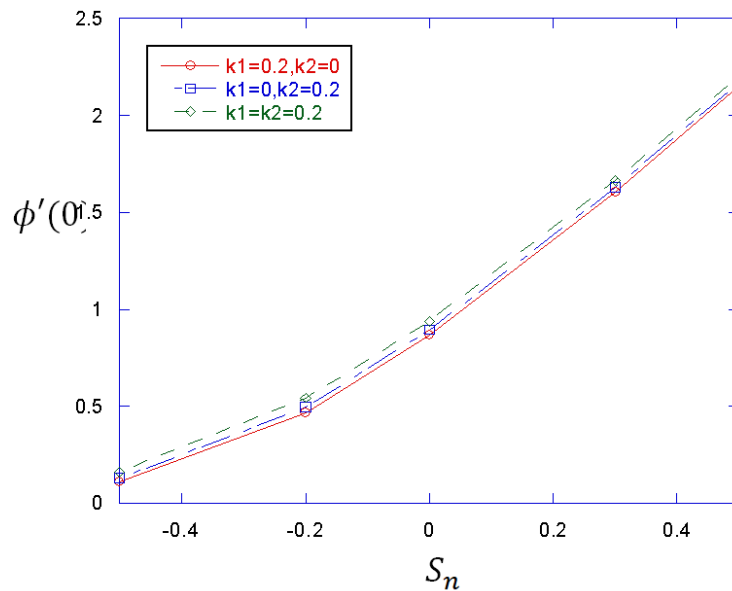


Fig 7: Mass transfer rate vs S_n
 when $N_t = N_b = 0.1, Le = 10, Pr = 6$

Fig. 2, 3 and 4 illustrate the graphical representation of the velocity profiles in the three porous media considered. (i.e. Darcy flow, non-Darcy flow and Darcy-Forchheimer flow). In all the velocity curves in the media, the rate of transport increases with increasing boundary layer thickness η , and approaches 0.9999 asymptotically at a boundary layer thickness of approximately $\eta = 6$. An increase in injection velocity leads to an increase in the fluid velocity, while suction does the opposite in all the three fluid flow regimes. Maximum velocity steepness is found to be highest in a Darcy-Forchheimer flow than in a non-Darcy flow and a Darcy flow, this is a result of the non-linear kinetic energy term present in a Darcy-Forchheimer flow. Since Darcy's law applies to creeping fluids (slow moving fluids), an increase in suction/injection is much noticeable or has much higher effect on the velocity distribution in a Darcy flow than in the other porous media (non-Darcy flow and Darcy-Forchheimer flow), since this regimes moves with much higher energy than a Darcy flow.

Fig. 5 Shows the effect of suction/injection velocity on the skin friction. It is observed that skin friction increases with an increase in Injection while Suction causes a decrease. It is recorded that its effect on the skin friction is found to be highest in a Darcy-Forchheimer flow than in a non-Darcy and a Darcy flow. Fig. 6 depicts suction/injection effect on heat transfer in the porous media. It is noticed from the figure

that heat transfer increases with an increase in injection and decreases with increasing Suction velocity. Heat transfer rate is found to be highest in a Darcy-Forchheimer flow when compared to the other media. Fig. 7 presents graphically the effect of suction/injection velocity on the mass transfer. Mass transfer also increases with increasing Injection and decreases with increasing Suction velocity. It is clearly observed that Mass transfer increase is highest in a Darcy-Forchheimer flow than in the other media.

4. CONCLUSION

In this paper, a combined similarity-numerical solution of suction/injection effect on flow, heat and mass transfer in porous media was studied. It was concluded that a Darcy-Forchheimer flow has the steepest velocity curves due to non-linearity, and that suction/injection effect is more pronounce on the velocity distribution of a creeping flow (Darcy flow) than the other media. The Darcy-Forchheimer flow is observed to have higher Skin friction, Heat and Mass transfer rate when compared to the other media.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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