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COVID-19 SIQRV Fractional-Order Mathematical Model with Vaccination and Quarantine Control Measures

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this study, an epidemic disease fractional-order mathematical model for Omicron, denoted as B.1.1.529 SARS-Cov-2 Variant, is constructed. Covid-19 vaccines and quarantine are considered here to ensure the host population's safety across the model. The fundamentals of positivity and boundedness in this model have been investigated and validated. The reproduction number was calculated to determine whether or not the disease would spread further in Tamilnadu. Infection-free steady-state solutions that exist are asymptotically stable locally and globally when $R_0 < 1$. Infection-present steady-state solutions also that are locally stable are discovered when $R_0 < 1$. Finally, the current Omicron variant pandemic data from Tamilnadu, India, is validated.

Keywords: Omicron; quarantine; vaccination; reproduction number; steady states; fractional derivative.

2020 Mathematics Subject Classification: 26A33, 34D20, 37N25.

1 Introduction

Omicron has been found in several nations as of November 24, 2021, and was still the dominant variety everywhere. Omicron may cause a milder form of disease, according to preliminary research, but some people who acquire this variant's infection still run the risk of developing serious illness, needing hospitalisation, and even passing away. You should take precautions even if only a tiny portion of those with Omicron infection require hospitalisation because the high incidence of cases could collapse the healthcare system. The Omicron variation, also known as B.1.1.5 29 SARS-Cov-2 Variant, was less contagious than the original COVID-19 virus and the Delta variant. People who have the Omicron variant infected may exhibit symptoms that are similar to those of earlier forms. At the most effective public health measure for preventing COVID-19 and reducing the possibility of new variations arising is still COVID-19 vaccination. This covers the initial course, booster shots, and any necessary subsequent doses. It is anticipated that the current vaccines will guard against the serious sickness, hospital stays, and fatalities brought on by Omicron variant infection. Breakthrough infections in vaccinated people, on the other hand, are likely.

Mathematical models have been developed in a wide range of ways to describe how diseases spread in subpopulations compartments. The highly infectious Omicron variant is challenging mathematicians to rethink models that have aided India's comprehension of COVID-19 and response to the outbreak. With the next wave of the pandemic, everyone from those who get tested to whome most likely to get the virus has altered, providing new hurdles for those who model its impact. The vaccine class is remembered for the model definition of an exemplary model, which allows appropriate antibodies to be supplied to the recovered and powerless persons in the host population when a wiped out individual recovers from an illness ([1], [2], [3]). Asymptotic strong characteristics move from the infection absent consistent state to the disease present consistent state. Because there are no known numerical approaches for developing Lyapunov capabilities for epidemic models, investigating the global aspects of a pestilence model framework is difficult ([4], [5]). Many mathematical models have been constructed for COVID-19 without the vaccination and Quarantine compartment ([6],[7],[8],[9]). In this study, we combined the results of papers ([10],[11],[12],[13], [14], [15],[16],[17],[18],[19],[20],[21]) to create an COVID-19 model with a variable population size.

The fractional differential has been utilised in the mathematical modelling of biological phenomena throughout the last few decades. This is due to the fact that fractional calculus can more precisely explain and process the retention and heritage characteristics of different materials than integer order models [22, 23, 24, 25, 26, 27]. As a result, many approaches have been used to examine the aforementioned field, including qualitative theory and numerical analysis. Fractional Mathematical models are effective instruments for researching infectious diseases. Recently, some authors looked at COVID-19 mathematical models using fractional order derivatives, and the findings were excellent. By reffering ([28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38]), fractional Omicron

mathematical model has been constructed and the existence, uniqueness, and positivity of the solution are also deduced. In this model the rate at which a given percentage of susceptible people is quarantined, is included here. There is also a compartment for vaccinations. Computational simulations were done at the end of the study to validate and reinforce our theoretical findings for Omicron B.1.1.529 SARS-Cov-2.

2 Model Formulation

In this section some important definitions and lemma has been given. The formulation of the model is discussed.

Definition 1. *Caputo fractional derivative [39]*

Let Ψ be a continuous function on [0, T]. The Caputo fractional derivative of order δ is given by

$$^{C}D^{\delta}\Psi(t) = \frac{1}{\Gamma(n-\delta)}\int_0^t (t-\alpha)^{n-\delta-1}\frac{d^n}{d\alpha^n}\Psi(t)(\alpha)d\alpha, \tag{2.1}$$

where $n = [\delta] + 1$ and $[\delta]$ represents the integer part function and $0 < \delta \leq 1$.

Definition 2. *Riemann-Liouville fractional integral [39]*

The Riemann-Liouville fractional integral of order δ is given by

$$\mathcal{I}^{\delta}\Psi(t) = \frac{1}{\Gamma(\delta)}\int_0^t (t-\alpha)^{\delta-1}\Psi(\alpha)d\alpha. \tag{2.2}$$

where $0 < \delta \leq 1$.

Definition 3. *Stability for Fractional-order Differential Equations*

Consider the fractional-order system

$$\begin{aligned} ^{C}D^{\delta}\mathcal{X}(t) &= \mathcal{J}(t,\mathcal{X}(t)), \quad 0 < \delta < 1, \\ \mathcal{X}(0) &= \mathcal{X}_0. \end{aligned} \tag{2.3}$$

Let $\mathcal{J}(t,\mathcal{X}_0)$ be the unique solution of the system (2.3) satisfying the initial condition $\mathcal{X}(0) = \mathcal{X}_0 \in \mathbb{R}^n$. Then:

1. *the trivial solution of (2.3) is said to be stable if for any $\epsilon > 0$, there exist $\delta = \delta(\epsilon) > 0$ such that, for every $\mathcal{X}_0 \in \mathbb{R}^n$ satisfying $\|\mathcal{X}_0\| < \delta$, we have $\|\mathcal{J}(t,\mathcal{X}_0)\| < \epsilon$ for any $t \geq 0$.*
2. *the trivial solution of (2.3) is said to be asymptotically stable if it is stable and there exists $\xi > 0$ such that $\lim_{t\to\infty}\mathcal{J}(t,\mathcal{X}_0) = 0$ for $\|\mathcal{X}_0\| < \xi$.*

A numerical Omicron mathematical model based on a consistent, non linear first request construction of common differential conditions is examined. The whole population $N(t)$ is subdivided into state factor sub-populations of people who are Susceptible individuals $S(t)$, Quarantined individuals $Q(t)$, Infected individuals $I(t)$, Recovered individuals $R(t)$ and Vaccinated individuals $V(t)$. The other parameters are given in Table 1.

In the model development, there are associated concerns that (i) the birth and passing rate is specific. (ii) People who are susceptible get contaminated if they come into contact with an infectious person who is not vaccinated. (iii) Vaccines lose their effectiveness over time, causing people to lose their immunity. (iv) A person who has been infected recovers after therapy. (v) No long-term recovery is possible. By the assumptions made, the system of equations of the model and network is formulated as

$$\begin{aligned}
\frac{dS}{dt} &= \Upsilon - \eta_1 S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\
\frac{dI}{dt} &= \eta_3 SI - (\eta_1 + \eta_6 + \eta_9 + \eta_{11})I, \\
\frac{dQ}{dt} &= \eta_6 I - (\eta_1 + \eta_4 + \eta_7)Q - \eta_8 QV, \\
\frac{dR}{dt} &= \eta_{11} I + \eta_7 Q - (\eta_1 + \eta_5 + \eta_{10})R, \\
\frac{dV}{dt} &= \eta_{10} R - \eta_2 SV - \eta_1 V + \eta_8 QV
\end{aligned} \tag{2.4}$$

with the initial conditions: $S(0) = S_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0^0, V(0) = V_0$.

Table 1. Parameters and their descriptions

Parameters	Descriptions
Υ	Rate at which humans are recruited into the population
η_1	The natural death rate applicable to all compartments
η_2	Rate at which a certain fraction of susceptible individuals receives vaccination
η_3	Effective infectious contact rate between the susceptible and infected individual
η_4	The quarantine rate of the susceptible individuals
η_5	The rate at which the recovered compartment loses its immunities to treatment
η_6	The rate at which the vaccinated compartment loses its immunities to vaccination
η_7	The treatment rate of the infected class
η_8	The natural recovery rates due to quarantine
η_9	The contact rate between Quarantined and Vaccinated people
η_{10}	The death rate induced by infections of infected individuals
η_{11}	The rate at which recovered individuals move to vaccinated compartment

Remark 1. *The system of equations can be written as*

$$\begin{aligned}
\frac{dS}{dt} &= \Upsilon - \eta_1 S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\
\frac{dI}{dt} &= \eta_3 SI - (\eta_{22})I, \\
\frac{dQ}{dt} &= \eta_6 I - \eta_{23} Q - \eta_8 QV, \\
\frac{dR}{dt} &= \eta_{11} I + \eta_7 Q - \eta_{24} R, \\
\frac{dV}{dt} &= \eta_{10} R - \eta_2 SV - \eta_{25} V + \eta_8 QV
\end{aligned} \tag{2.5}$$

where $\eta_{21} = \eta_1$, $\eta_{22} = \eta_1 + \eta_6 + \eta_9 + \eta_{11}$, $\eta_{23} = \eta_1 + \eta_4 + \eta_7$, $\eta_{24} = \eta_1 + \eta_5 + \eta_{10}$, *and* $\eta_{25} = \eta_1$
Subject to initial conditions: $S(0) = S_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0^0, V(0) = V_0$.

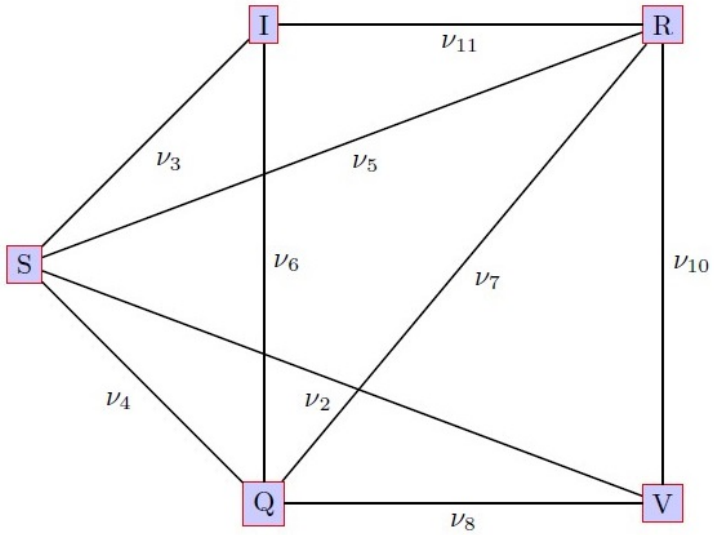

Fig. 1. Network of the Fractional-order SIQRV Model

2.1 FDE model formulation

The corresponding system of fractional order differential equations model can be written as

$$\begin{aligned} {}^{C}D^{\delta}S &= \Upsilon - \eta_{21}S + \eta_2 SV - \eta_3 SI + \eta_4 Q + \eta_5 R \\ {}^{C}D^{\delta}I &= \eta_3 SI - (\eta_{22})I, \\ {}^{C}D^{\delta}Q &= \eta_6 I - \eta_{23}Q - \eta_8 QV, \\ {}^{C}D^{\delta}R &= \eta_{11} I + \eta_7 Q - \eta_{24} R, \\ {}^{C}D^{\delta}V &= \eta_{10} R - \eta_2 SV - \eta_{25} V + \eta_8 QV \end{aligned} \tag{2.6}$$

Subject to initial conditions: $S(0) = S_0, Q(0) = Q_0, I(0) = I_0, R(0) = R_0^0, V(0) = V_0$.

The system (2.6) can be written in the form:

$$\begin{aligned} {}^{C}D^{\delta}\mathcal{X}(t) &= \mathcal{J}(t, \mathcal{X}(t)), \quad 0 < \delta \leq 1, \\ \mathcal{X}(0) &= \mathcal{X}_0. \end{aligned} \tag{2.7}$$

To find the existence of solution, a Banach space is defined as $\mathcal{B} = B_1 \times B_2 \times B_3 \times B_4 \times B_5$, where $B_i = C([0,T]), (i = 1, 2, \ldots, 5)$ under the norm

$$\|\mathcal{X}\| = \| (S, I, Q, R, V) \| = \max_{t\in[0,t]} [|S(t)|, |I(t)|, |Q(t)|, |R(t)|, |V(t)|].$$

Let $\mathcal{Y} : \mathcal{A} \to \mathcal{A}$ be an operator defined as follows:

$$\mathcal{Y}(\mathcal{X})(t) = Y_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, \mathcal{X}(\gamma)) d\gamma.$$

Using the Riemann Liouville type integral, equation (2.7) solved as follows: $\mathcal{X}(t) = \mathcal{X}_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, \mathcal{X}(\gamma)) d\gamma$ where

$$\begin{cases} S(t) = S_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, S) d\gamma, \\ I(t) = I_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, I) d\gamma, \\ Q(t) = Q_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, Q) d\gamma, \\ R(t) = R_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, R) d\gamma, \\ V(t) = V_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, V) d\gamma, \end{cases} \tag{2.8}$$

with

$$\begin{cases} \mathcal{J}(\gamma, S) = \Upsilon - \eta_{21} S - \eta_3 SI + \eta_4 Q + \eta_5 R \\ \mathcal{J}(\gamma, I) = \eta_3 SI - (\eta_{22}) I, \\ \mathcal{J}(\gamma, Q) = \eta_6 I - \eta_{23} Q - \eta_8 QV, \\ \mathcal{J}(\gamma, R) = \eta_{11} I + \eta_7 Q - \eta_{24} R, \\ \mathcal{J}(\gamma, V) = \eta_{10} R - \eta_2 SV - \eta_{25} V + \eta_8 QV \end{cases} \tag{2.9}$$

2.2 Positivity and existence of solution

To investigate the non-negativity of the solution, we define

$R_+^5 = \{\mathcal{X} \in R^5 \mid \mathcal{X} \geq 0\}$ and $\mathcal{X}(t) = (S(t), Q(t), I(t), R(t), V(t))^T$

Now we remind the generalized mean values theorem [27].

Lemma 1. *Let* $\mathcal{X}(\mathrm{t}) \in \mathrm{C[c, d]}$ *and* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \in (\mathrm{c, d}]$*, then* $\mathcal{X}(t) = \mathcal{X}(c) + \frac{1}{\Gamma(\beta)} \left({}^C D_t^\delta \mathcal{X}\right)(\zeta)(t-c)^\delta$ *with* $c \leq \zeta \leq t, \forall t \in (c, d]$.

Corollary 1. *Let* $\mathcal{X}(\mathrm{t}) \in C[\mathrm{c, d}]$ *and* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \in (\mathrm{c, d}]$ *where* $\delta \in (0, 1]$*. Then, it is clear from lemma 1 that if* ${}^C D_t^\delta \mathcal{X}(\mathrm{t}) \geq 0, \forall \mathrm{t} \in (\mathrm{c, d}]$*, then the function* $\mathcal{X}(\mathrm{t})$ *is non-decreasing and if* ${}^C D_\mathrm{t}^\delta \mathcal{X}(\mathrm{t}) \leq 0, \forall \mathrm{t} \in (\mathrm{c, d}]$*, then the function* $\mathcal{U}(\mathrm{t})$ *is non-increasing for all* $t \in [c, d]$.

Now to prove the non-negativity of the solution, it is necessary to investigate that the solution on every hyperplane bounding the positive orthant, the vector field points to R_+^5.

Theorem 1. *If S(0), I(0), Q(0), R(0), V(0) are positive and bounded in* R_+^5*, then S(t), I(t), Q(t), R(t), V(t) are also positive and bounded in* R_+^5 *for all* $t > 0$.

Proof. From model (2.6), we get

$$\begin{aligned} {}^C D_t^\delta S(t)_{S=0} &= \Upsilon + \eta_4 Q + \eta_5 R \geq 0 \\ {}^C D_t^\delta I(t)_{I=0} &= 0 \geq 0 \\ {}^C D_t^\delta Q(t)_{Q=0} &= \eta_6 I \geq 0 \\ {}^C D_t^\delta R(t)_{R=0} &= \eta_{11} I + \eta_7 Q \geq 0 \\ {}^C D_t^\delta V(t)_{V=0} &= \eta_{10} R \geq 0 \end{aligned}$$

Hence, by using the corollary 1 we get that our solution is nonnegative and will lie in the given feasible region.

Adding all the equations of the system in (2.6), we get

$${}^C D_t^\delta N = \Upsilon - \eta_1 (S + I + Q + R + V) - \eta_{10} I \tag{2.10}$$

and in the infection free state we have ${}^C D_t^\delta N = \Upsilon - N\eta_1$. Thus by taking Laplace transform we get

$$N(s) = \frac{\Upsilon}{s(s^\delta + \eta_1)} + N(0) \frac{s^{(\delta-1)}}{s^\delta + \eta_1}, \tag{2.11}$$

By taking inverse Laplace transform and solving we get

$$\lim_{t\to\infty} N(t) \leq \frac{\Upsilon}{\eta_1}. \tag{2.12}$$

Then it follows the positivity and bounded for all t>0. □

The following theorems show the existence of a solution.

Theorem 2. *Let $S(t), I(t), Q(t), R(t)$, and $V(t)$ be nonnegative bounded functions. Then the system (2.9) satisfies Lipschitz cindition.*

Proof. Assume that $S(t), I(t), Q(t), R(t)$, and $V(t)$ are nonnegative bounded functions. That is, there are some positive constants $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$, such that

$\|S(t)\| \leq \xi_1, \|I(t)\| \leq \xi_2, \|Q(t)\| \leq \xi_3, \|R(t)\| \leq \xi_4, \|V(t)\| \leq \xi_5$.

Consider the function $\mathcal{J}(\gamma, S)$, for any S and S_1, we can get

$$\begin{aligned}
\|\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_1)\| &= \|\eta_1 (S - S_1) + \eta_2 V (S_1 - S) + \eta_3 I(S - S_1)\| \\
&\leq \|\eta_1 (S - S_1)\| + \|\eta_2 V (S - S_1)\| + \|\eta_3 I (S - S_1)\| \\
&\leq (\eta_1 + \eta_2 \|V(t)\| + \eta_3 \|I(t)\|) \|S - S_1\| \\
&\leq (\eta_1 + \eta_2 \xi_5 + \eta_3 \xi_2) \|S - S_1\| \\
&\leq \mathcal{G}_{\mathcal{J}_1} \|S - S_1\|
\end{aligned} \tag{2.13}$$

where $\mathcal{G}_{\mathcal{J}_1} = \eta_1 + \eta_2 \xi_5 + \eta_3 \xi_2$. Hence, $\mathcal{J}(\gamma, S)$ satisfies the Lipschitz condition. Similarly, we can find $\mathcal{G}_{\mathcal{J}_i}$, for $i = 2, 3, 4, 5$ so that $\mathcal{J}(\gamma, S), \mathcal{J}(\gamma, Q), \mathcal{J}(\gamma, I), \mathcal{J}(\gamma, R)$, and $\mathcal{J}(\gamma, V)$ satisfy the Lipschitz's conditions. □

Consider the equation (2.8), it can be formulated as :

$$X_n(t) = \begin{cases}
S_n(t) = S_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, S_{n-1}) d\gamma, \\
I_n(t) = I_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, I_{n-1}) d\gamma, \\
Q_n(t) = Q_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, Q_{n-1}) d\gamma, \\
R_n(t) = R_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, R_{n-1}) d\gamma, \\
V_n(t) = V_0 + \frac{1}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, V_{n-1}) d\gamma.
\end{cases}$$

The provided initial conditions establish the first elements of the given equations. The contrast between two terms is expressed as :

$$\begin{aligned}
\Psi_{1_n}(t) = S_n(t) - S_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S_{n-1}) - \mathcal{J}(\gamma, S_{n-2})] \, d\gamma, \\
\Psi_{2_n}(t) = I_n(t) - I_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, I_{n-1}) - \mathcal{J}(\gamma, I_{n-2})] \, d\gamma, \\
\Psi_{3_n}(t) = Q_n(t) - Q_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, Q_{n-1}) - \mathcal{J}(\gamma, Q_{n-2})] \, d\gamma, \\
\Psi_{4_n}(t) = R_n(t) - R_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, R_{n-1}) - \mathcal{J}(\gamma, R_{n-2})] \, d\gamma, \\
\Psi_{5_n}(t) = V_n(t) - V_{n-1}(t) &= \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, V_{n-1}) - \mathcal{J}(\gamma, V_{n-2})] \, d\gamma
\end{aligned}$$

where

$$X_n(t) = \begin{cases}
S_n(t) = \sum_{i=0}^n \Psi_{1_i}(t), \\
I_n(t) = \sum_{i=0}^n \Psi_{2_i}(t), \\
Q_n(t) = \sum_{i=0}^n \Psi_{3_i}(t), \\
R_n(t) = \sum_{i=0}^n \Psi_{4_i}(t), \\
V_n(t) = \sum_{i=0}^n \Psi_{5_i}(t).
\end{cases} \tag{2.14}$$

Consider

$$\|\Psi_{1_n}(t)\| = \|S_n(t) - S_{n-1}(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S_{n-1}) - \mathcal{J}(\gamma, S_{n-1})]\, d\gamma$$
$$= \frac{\xi_1}{\Gamma(\delta)} \int_0^t \|S_{n-1} - S_{n-2}\|\, d\gamma = \frac{\xi_1}{\Gamma(\delta)} \int_0^t \left\|\Psi_{1_{n-1}}(t)\right\| d\gamma$$

Hence, we can get

$$\|\Psi_{i_n}(t)\| = \frac{\xi_i}{\Gamma(\delta)} \int_0^t \left\|\Psi_{i_{n-1}}(t)\right\| d\gamma \ for\ i = 1, 2, \ldots 5. \tag{2.15}$$

Now the functions defined in (2.14) are exist and smooth. For,

We have that the functions $S(t), I(t), Q(t), R(t)$, and $V(t)$ are bounded and all kernels $\mathcal{J}(t, S), \mathcal{J}(t, I), \mathcal{J}(t, Q), \mathcal{J}(t, R),\ and\ \mathcal{J}(t, V)$, fulfill Lipschitz's conditions, thus, we obtain the following relations:

$$\begin{cases} \|\Psi_{1_n}(t)\| \leq \|S(0)\| \left\| \frac{\xi_1}{\Gamma(\delta)} t \right\|^n, \\ \|\Psi_{2_n}(t)\| \leq \|I(0)\| \left\| \frac{\xi_2}{\Gamma(\delta)} t \right\|^n, \\ \|\Psi_{3_n}(t)\| \leq \|Q(0)\| \left\| \frac{\xi_3}{\Gamma(\delta)} t \right\|^n, \quad . \\ \|\Psi_{4_n}(t)\| \leq \|R(0)\| \left\| \frac{\xi_4}{\Gamma(\delta)} t \right\|^n, \\ \|\Psi_{5_n}(t)\| \leq \|V(0)\| \left\| \frac{\xi_5}{\Gamma(\delta)} t \right\|^n. \end{cases} \tag{2.16}$$

The system (2.16) shows the existence and smoothness of the function defined in (2.15).

Theorem 3. *Let $\mathcal{Y} : \mathcal{A} \to \mathcal{A}$ be completely continuous and let $\mathcal{J} : [0, T] \times \mathcal{A} \to \mathbb{R}$ is continuous and there exists a constant $\mathcal{G}_{\mathcal{J}} > 0$ such that for $X, X_1 \in \mathcal{A}$,*

$$|\mathcal{J}(t, X) - \mathcal{J}(t, X_1)| \leq \mathcal{G}_{\mathcal{J}} |X - X_1|$$

is hold. Then there is at least one solution for the considered system (2.6).

Proof. To prove the operator $\mathcal{Y}$ is completely continuous. The sequence $\{X_n\}$ converges to $X \in \mathcal{A}$. For, after n-iterations define the remainder terms as $D_{1_n}(t), D_{2_n}(t), D_{3_n}(t), D_{4_n}(t), D_{5_n}(t)$, such that

$$\begin{aligned} S(t) - S(0) &= S_n(t) - D_{1_n}(t), \\ I(t) - I(0) &= I_n(t) - D_{2_n}(t), \\ Q(t) - Q(0) &= Q_n(t) - D_{3_n}(t), \\ R(t) - R(0) &= R_n(t) - D_{4_n}(t), \\ V(t) - V(0) &= V_n(t) - D_{5_n}(t). \end{aligned}$$

Using triangle inequality along with the Lipschitz condition of $\mathcal{J}(t, S)$, we obtain:

$$\|D_{1_n}(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_{n-1})]\, d\gamma \leq \frac{\xi_1}{\Gamma(\delta)} \|S - S_{n-1}\|\, t.$$

Applying the above process recursively, we get

$$\|D_{1_n}(t)\| \leq \left\| \frac{C_1}{\Gamma(\delta)} t \right\|^{n+1} \xi_1.$$

Then, at t_0

$$\|D_n(t)\| \leq \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{n+1} \xi_1.$$

Taking limit as n tends to infinity, we get

$$\lim_{n\to\infty} \|D_{1_n}(t)\| \leq \lim_{n\to\infty} \left\| \frac{C_1}{\Gamma(\delta)} t_0 \right\|^{n+1} \xi_1. \tag{2.17}$$

For $\frac{C_i}{\Gamma(\delta)} t_0 < 1$, Equation (2.17) becomes $\lim_{n\to\infty} \|D_{1_n}(t)\| = 0$ Similarly, as n tends to infinity, we can get $\|D_{i_n}(t)\| \to 0$.

Hence for $t \in [0, T]$, we have $S_n(t) \to S(t)$ *as* $n \to \infty$

$$\begin{aligned}\|\mathcal{Y}(S_n) - \mathcal{Y}(S)\| &\leq \frac{1}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t (t-\gamma)^{\delta-1} - |\mathcal{J}(\gamma, S_n(\gamma))\mathcal{J}(\gamma, S(\gamma))| d\gamma, \\ &\leq \frac{\mathcal{G}_{\mathcal{J}}}{\Gamma(\delta)} \|S_n - S\|_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{G}_{\mathcal{J}}}{\Gamma(\delta+1)} \|S_n - S\|.\end{aligned}$$

Since, $S_n \to S$, so $\|\mathcal{Y}(S_n) - \mathcal{Y}(S)\| \to 0$ as $n \to \infty$ and hence $\|\mathcal{Y}(X_n) - \mathcal{Y}(X)\| \to 0$ as $n \to \infty$. Thus $\mathcal{Y}$ is continues. Let a bounded set $\mathcal{M} \subset \mathcal{A}$. Then by definition of $\mathcal{A}$, $|\mathcal{J}(\mathrm{t}, X(t))| \leq \mathcal{L}_{\mathcal{J}}, \mathcal{L}_{\mathcal{J}} > 0, \forall X \in \mathcal{M}$. Then for each $X \in \mathcal{M}$, we can obtain

$$\begin{aligned}\|\mathcal{Y}(X)\| &\leq \frac{1}{\Gamma(\delta)} \max_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} |\mathcal{J}(\gamma, Y(\gamma))| d\gamma \\ &\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{t\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)}\end{aligned}$$

Thus, $\mathcal{Y}$ is uniformly bounded. Further suppose $0 \leq t_2 \leq t_1 \leq T$. Then

$$\begin{aligned}\|\mathcal{Y}(X)(t_1) - \mathcal{Y}(X)(t_2)\| &\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{t\to[0,T]} | \int_0^{t_1} (t_1-\gamma)^{\delta-1} d\gamma - \int_0^{t_2} (t_2-\gamma)^{\delta-1} d\gamma|, \\ &\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)} max_{t\to[0,T]} |t_1^\delta - t_2^\delta| \to 0 \ as \ t_1 \to t_2.\end{aligned}$$

Thus, $\mathcal{Y}$ is equicontinuous. $\mathcal{Y}$ is compact and hence it is completely continuous because of the continuousness and boundedness of it . Let $\Psi = \{X \in \mathcal{A} : X = \rho\mathcal{Y}(X), \rho \in [0, 1]\}$, we need to confirm that Ψ is bounded. Suppose $X \in \Psi$, say S then for $t \in [0,$ T$]$, we have:

$$\begin{aligned}\|S\| &= max_{t\to[0,T]} \{\frac{\rho}{\Gamma(\delta)} \int_0^t (t-\gamma)^{\delta-1} \mathcal{J}(\gamma, S(\gamma)) d\gamma\} \\ &\leq \frac{\mathcal{L}_{\mathcal{J}}}{\Gamma(\delta)} \max_{T\to[0,T]} \int_0^t (t-\gamma)^{\delta-1} d\gamma \leq \frac{T^\delta \mathcal{L}_{\mathcal{J}}}{\Gamma(\delta+1)}.\end{aligned}$$

Thus the operator is completely continuous. The set Ψ is also bounded. Therefore, $\mathcal{Y}$ has at least one fixed point[40]. So, the considered system (2.6) has the same number of solutions. □

Theorem 4. *If* $\frac{\xi_i}{\Gamma(\delta)} t < 1$*, for* $i = 1, 2, \cdots, 5$*, then the system (2.6) has a unique solution.*

Proof. . Assume that $\{S_\nu(t), I_\nu(t), Q_\nu(t), R_\nu(t), V_\nu(t)\}$ is another set of solutions of system (2.6) then,

$$\|S(t) - S_\nu(t)\| = \frac{1}{\Gamma(\delta)} \int_0^t [\mathcal{J}(\gamma, S) - \mathcal{J}(\gamma, S_\nu)] d\gamma = \frac{\xi_1}{\Gamma(\delta)} t \|S(t) - S_\nu(t)\|$$

Thus

$$(1 - \frac{\xi_1}{\Gamma(\delta)} t)\|S(t) - S_1(t)\| \leq 0. \tag{2.18}$$

Since $\frac{\xi_i}{\Gamma(\delta)} t < 1$ for i = 1, (2.18) becomes $\|S(t) - S_\nu(t)\| = 0$
Hence $S(t) = S_\nu(t)$. Similarly, for $i = 2, 3, 4$ *and* 5, we can get $I(t) = I_\nu(t)$; $Q(t) = Q_\nu(t)$; $R(t) = R_\nu(t)$; $V_\nu(t) = V_\nu(t)$. Hence the system has unique (2.6) solution. □

2.3 Existence of steady state solutions

The steady-state solutions exist as a result of research into the long-term behaviour of which is heavily dependent on R_0 and its steady-state solutions. The model under consideration in this paper has two steady-state solutions. The model system (1) is made static, i.e. the time-independent solutions are obtained.

The steady-state solutions in the absence of infections i.e., $I = 0$ is given by

$$\begin{aligned} E^0 &= (S, I, Q, R, V) \\ &= \big(\frac{\Gamma}{\eta_1}, 0, 0, 0, 0\big) \end{aligned} \tag{2.19}$$

Also, the steady-state solutions when infection is persistent i.e., $I \neq 0$ is given by,

$$\begin{aligned} E^* &= (S^*, I^*, Q^*, R^*, V^*) \\ &= \big(\frac{\eta_{22}}{\eta_3}, \frac{\eta_1\eta_{22}(R_0-1)}{\eta_3(\eta_3\eta_{22}-\eta_4 A+\eta_5 B-\eta_2\eta_{22}C)}, \\ &\quad \frac{\eta_6 I^*}{\eta_{23}+\eta_8 V^*}, \frac{(\eta_6\eta_7+\eta_{11}[\eta_{23}+\eta_8 V^*])I^*}{\eta_{24}(\eta_{23}+\eta_8 V^*)}, \\ &\quad \frac{\eta_{10}\eta_3 B I^*}{\eta_2\eta_{22}+\eta_1\eta_3+\eta_1\eta_8 Q^*}\big) \end{aligned} \tag{2.20}$$

where,
$A = \frac{\eta_6}{\eta_{23}+\eta_8 V^*}, B = \frac{(\eta_6\eta_7+\eta_{11}[\eta_{23}+\eta_8 V^*])}{\eta_{24}(\eta_{23}+\eta_8 V^*)}, C = \frac{\eta_{10}\eta_3 B}{\eta_2\eta_{22}+\eta_1\eta_3+\eta_1\eta_8 Q^*}$
The fundamental reproduction number R_0 is from the next generation method ([41], [42]) as follows:

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_3 S & 0 & 0 & 0 \\ 0 & \eta_6 & 0 & 0 & 0 \\ 0 & \eta_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \eta_1 & 0 & \eta_4 & \eta_5 & 0 \\ 0 & \eta_1+\eta_9+\eta_{11}+\eta_6 & 0 & 0 & 0 \\ 0 & \eta_6 & \eta_1+\eta_4+\eta_7 & 0 & 0 \\ 0 & \eta_{11} & \eta_7 & \eta_1+\eta_5+\eta_{10} & 0 \\ 0 & 0 & 0 & 0 & \eta_1 \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\eta_1} & \frac{\eta_5(\eta_{11}\eta_{23}-\eta_6\eta_{24})+\eta_4\eta_6\eta_{24}}{\eta_1\eta_{22}\eta_{23}\eta_{24}} & \frac{\eta_5\eta_7-\eta_4\eta_{24}}{\eta_1\eta_{23}\eta_{24}} & -\frac{\eta_5}{\eta_1\eta_{24}} & 0 \\ 0 & \frac{1}{\eta_{22}} & 0 & 0 & 0 \\ 0 & -\frac{\eta_6}{\eta_{22}\eta_{23}} & \frac{1}{\eta_{23}} & 0 & 0 \\ 0 & \frac{\eta_{11}\eta_{23}+\eta_6\eta_7}{\eta_{22}\eta_{23}\eta_{24}} & -\frac{\eta_7}{\eta_{23}\eta_{24}} & \frac{1}{\eta_{24}} & 0 \\ 0 & \frac{\eta_{10}(\eta_{11}\eta_{23}-\eta_6\eta_7)}{\eta_{22}\eta_{23}\eta_{24}} & \frac{\eta_7\eta_{10}}{\eta_1\eta_{23}\eta_{24}} & \frac{\eta_{10}}{\eta_1\eta_{24}} & \frac{1}{\eta_1} \end{pmatrix}$$

Then, R_0 is the largest eigenvalue of the spectral radius given by

$$R_0 = \rho(FV^{-1}) = \eta_3\big(\frac{\Upsilon}{\eta_1(\eta_1+\eta_6+\eta_9+\eta_{11})}\big) \tag{2.21}$$

When $R_0 < 1$, the infections vanish out of the host population. But if $R_0 > 1$, then the infections ravage and becomes endemic, which calls for appropriate medical interventions to stop the disease spread.

3 Stability Analysis

Theorem 5. *The disease-free equilibrium E_0 of the given system (2.6) is stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. To state the stability analysis of the disease-free equilibrium points, we analyse the linearization of the given system (2.6) at any equilibrium point $(S^*, I^*, Q^*, R^*, V^*)$ as follows:

$$\begin{aligned}
{}^{C}D^{\delta}S &= \Upsilon - \eta_{21}S + \eta_2 S^*V + \eta_2 SV^* - \eta_3 S^*I - \eta_3 SI^* + \eta_4 Q + \eta_5 R \\
{}^{C}D^{\delta}I &= \eta_3 S^*I + \eta_3 SI^* - \eta_{22}I, \\
{}^{C}D^{\delta}Q &= \eta_6 I - \eta_{23}Q - \eta_8 Q^*V - \eta_8 QV^*, \\
{}^{C}D^{\delta}R &= \eta_{11}I + \eta_7 Q - \eta_{24}R, \\
{}^{C}D^{\delta}V &= \eta_{10}R - \eta_2 S^*V - \eta_2 SV^* - \eta_{25}V + \eta_8 Q^*V + \eta_8 QV^*
\end{aligned} \tag{3.1}$$

Applying the Laplace transform on both sides of above system (3.1) gives

$$\begin{aligned}
s^{\delta}\mathcal{L}[S(s)] - s^{\delta-1}S(0) &= \Upsilon - \eta_1\mathcal{L}[S(s)] + \eta_2 S^*\mathcal{L}[V(s)] + \eta_2 V^*\mathcal{L}[S(s)] \\
&\quad -\eta_3 S^*\mathcal{L}[I(s)] - \eta_3 I^*\mathcal{L}[S(s)] + \eta_4 Q + \eta_5 R \\
s^{\delta}\mathcal{L}[I(s)] - s^{\delta-1}I(0) &= \eta_3 S^*\mathcal{L}[I(s)] + \eta_3 I^*\mathcal{L}[S(s)] - \eta_{22}\mathcal{L}[I(s)], \\
s^{\delta}\mathcal{L}[Q(s)] - s^{\delta-1}Q(0) &= \eta_6\mathcal{L}[I(s)] - \eta_{23}\mathcal{L}[Q(s)] - \eta_8 Q^*\mathcal{L}[V(s)] \\
&\quad -\eta_8 V^*\mathcal{L}[Q(s)], \\
s^{\delta}\mathcal{L}[R(s)] - s^{\delta-1}R(0) &= \eta_{11}\mathcal{L}[I(s)] + \eta_7\mathcal{L}[Q(s)] - \eta_{24}\mathcal{L}[R(s)], \\
s^{\delta}\mathcal{L}[V(s)] - s^{\delta-1}V(0) &= \eta_{10}\mathcal{L}[R(s)] - \eta_2 S^*\mathcal{L}[V(s)] - \eta_2 V^*\mathcal{L}[S(s)] \\
&\quad -\eta_{25}\mathcal{L}[V(s)] + \eta_8 Q^*\mathcal{L}[V(s)] + \eta_8 V^*\mathcal{L}[Q(s)]
\end{aligned} \tag{3.2}$$

where $\mathcal{L}[S(s)], \mathcal{L}[I(s)], \mathcal{L}[Q(s)], \mathcal{L}[R(s)]$, and $\mathcal{L}[I_(s)]$ are the Laplace transformations of $S(t), Q(t), I(t), R(t)$, and $V(t)$. The proposed system (3.2) can be rewritten by

$$\Delta(s) \cdot [L[S(s)]L[I(s)]L[Q(s)]L[R(s)]L[V(s)]] = [\nu_1(s)\nu_2(s)\nu_3(s)\nu_4(s)\nu_5(s)]$$

where

$$\begin{cases}
\nu_1(s) = s^{\delta-1}S(0) \\
\nu_2(s) = s^{\delta-1}I(0) \\
\nu_3(s) = s^{\delta-1}Q(0) \\
\nu_4(s) = s^{\delta-1}R(0) \\
\nu_5(s) = s^{\delta-1}V(0)
\end{cases}$$

Hence $\Delta(s) =$

$$\begin{bmatrix}
A_{11} & -\eta_3 S^* & \eta_4 & \eta_5 & \eta_2 S^* \\
\eta_3 I^* & A_{22} & 0 & 0 & 0 \\
0 & \eta_6 & s^{\delta} + \eta_{23} & 0 & -\eta_8 Q^* \\
0 & \eta_{11} & \eta_7 & s^{\delta} + \eta_{24} & 0 \\
-\eta_2 V^* & 0 & \eta_8 V^* & \eta_{10} & A_{55}
\end{bmatrix}$$

where $A_{11} = s^{\delta} + (\eta_1 + \eta_3 I^* + \eta_2 V^*)$, $A_{22} = s^{\delta} + \eta_{22} - \eta_3 S^*$ and $A_{55} = s^{\delta} + \eta_1 + \eta_2 S^* - \eta_8 Q^*$,

which is a characteristic matrix of system (3.2). Now, the characteristic matrix of the proposed system at disease-free equilibrium (DFE) (2.19) is given by $\Delta(s) =$

$$\begin{bmatrix}
s^{\delta} + \eta_1 & -\eta_3 S & \eta_4 & \eta_5 & \eta_2 S \\
0 & B_{22} & 0 & 0 & 0 \\
0 & \eta_6 & s^{\delta} + \eta_{23} & 0 & 0 \\
0 & \eta_{11} & \eta_7 & s^{\delta} + \eta_{24} & 0 \\
0 & 0 & 0 & \eta_{10} & B_{55}
\end{bmatrix}$$

where $B_{22} = s^{\delta} - \eta_3 S + \eta_{22}$ and $B_{55} = s^{\delta} + \eta_1 + \eta_2 S$.

Then from the Jacobian matrix, the characteristic polynomial is $(s^{\delta} + \eta_1)(s^{\delta} + (\eta_1 + \eta_4 + \eta_6 + \eta_7))(s^{\delta} + (\eta_1 + \eta_4 + \eta_{10}))(s^{\delta} + (-\eta_3 S - (\eta_1 + \eta_6 + \eta_9 + \eta_{11})))(s^{\delta} + (\eta_1 + \eta_5 + \eta_{10}))(s^{\delta} + (\eta_1 + \eta_2 S))$.

Now, the System (2.4) is stable iff $\eta_3 S - (\eta_1 + \eta_6 + \eta_9 + \eta_{11}) < 0$.

Hence $\eta_3\big(\frac{\Gamma}{\eta_1(\eta_1+\eta_6+\eta_9+\eta_{11})}\big) < 1$.

Then clearly the infection free steady state of (2.19) is locally asymptotically stable if $R_0 < 1$. □

4 Numerical Analysis

In the second wave of the Corona virus, India experienced a high infection rate. We've gathered data from Tamilnadu, India, for this article ([43]). Mathematica and Matlab are used to simulate the numerical solution. The values of the variables and parameters are listed in the Tables below 2 and 3.

Table 2. Variables in Model and their values

Variable	Descriptions	Values
$S(0)$	Susceptible individuals	30095
$Q(0)$	Quarantine Individuals	322
$I(0)$	Infected individuals	35
$R(0)$	Recovered individuals	51
$V\ (0)$	Vaccinated Individuals	42846

The solution for (2.6) demonstrates that it is unstable locally and will never become stable when $R_0 > 1$, as shown in the figures. The steady state solution becomes stable when the contact rate is controlled and the vaccination class is increased when $R_0 < 1$. We conclude from all of the data that if the number of isolated, recovered, and vaccinated people increases, the host community will be safe from the Omicron variant. We also discovered that if the intercessions are strictly followed, the spread of the second wave of SARS Cov-2 Omicron variant is reduced.

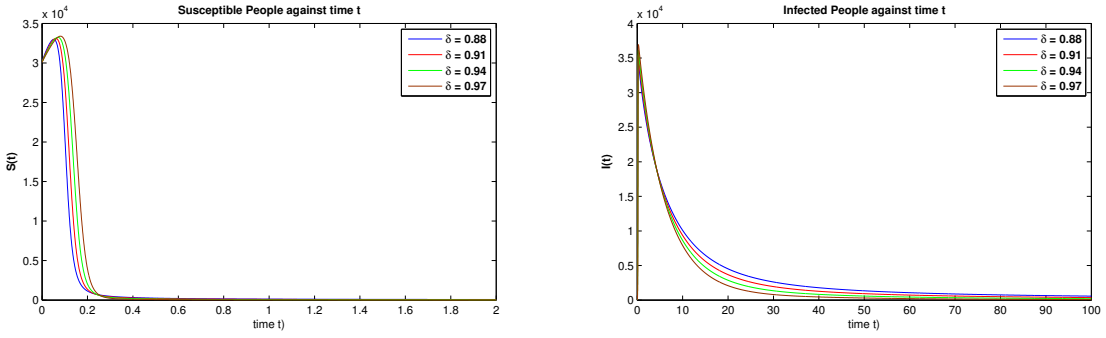

Fig. 2. Susceptible E(t) and Infected I(t) people against time t in the data of Tamilnadu

Fig. 2 describes the visualization of the impact of the Omicron variant in the Susceptible individual, Infected individual against time t in the overall state of Tamilnadu. People are becoming less susceptible to infection and less probable to obtain it as a result of increased vaccination and quarantine.

Table 3. Parameters in Model and their values

Parameters	Descriptions	Values
Υ	Per Capita Recruitment Rate	5
η_1	Natural death rate	0.065
η_2	Rate of vaccination of susceptible individuals	0.0109
η_3	Infectious rate	0.0012
η_4	Quarantine rate	0.0098
η_5	Rate of immunity loss	0.0017
η_6	Treatment rate	0.1087
η_7	recovery rate due to Quarantine	0.0146
η_8	The contact rate between Quarantined and Vaccinated people	0.0098
η_9	The death rate induced by infections of infected individuals	0.0006
η_{10}	The rate at which recovered individual moves to vaccinated compartment	0.92
η_{11}	The natural recovery rates transfere from infected to recovered individuals	0.045

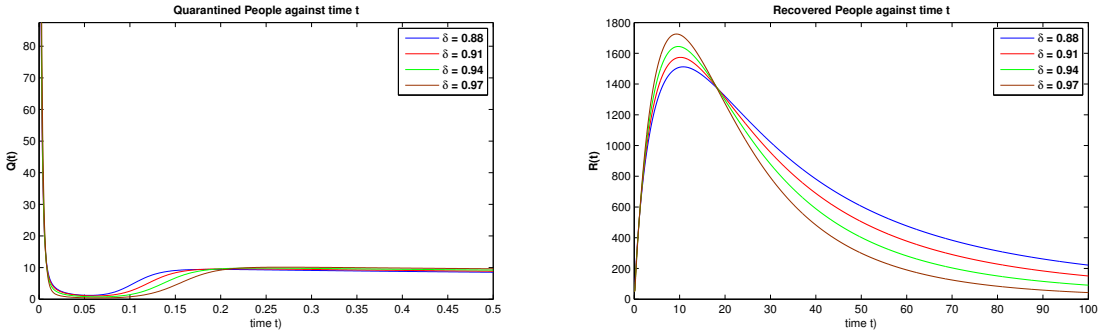

Fig. 3. Quarantined and recovered people against time t

Fig. 3 describes the Quarantined and recovered people of the host of human population respectively against the time t in the state of Tamilnadu. Figs. 2 and 3 show how persons in Tamilnadu were infected and confirmed with the Omicron variant in the beginning and recovered by the end of March 31st, 2022. It is obvious from Figs. 2 and 3 that once the infected population increases, all other compartments increase as well. In addition, if there is an increase in isolated and vaccinated classes, a disease-free field will exist after 80 days when δ=0.97.

According to Table 3, as of March 31, the number of infected people in Tamilnadu districts has decreased to low level with no death based on RT PCR sample tests.

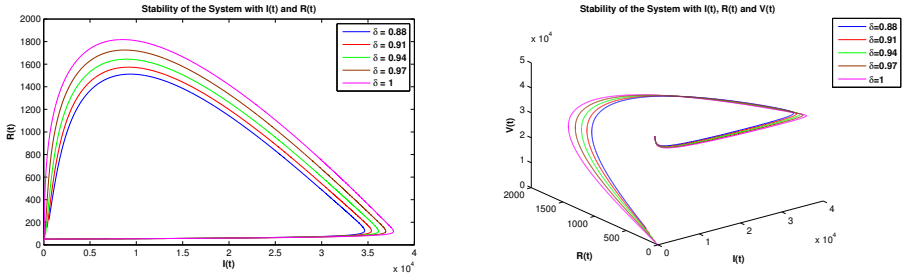

Fig. 4. Stability condition of the fractionsl model against time t with I(t) and R(t) and Stability condition against time t with I(t), R(t) and V(t) -3D Plot

The stability of the Omicron mathematical model for Tamilnadu (Fig. 4). During the Omicron period, which runs from December 25th to March 11th, 2022, the people of these four districts have a high rate of illness. When people were vaccinated according to government instructions, the infection rate progressively decreased to a low level.

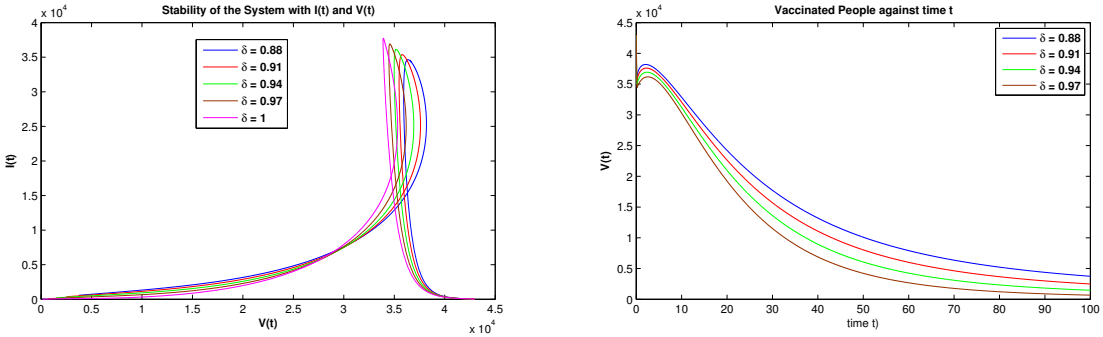

Fig. 5. Infected and Vaccinated against time t in Tamilnadu

Fig. 5 describes the relation between the rates of infected, isolated and vaccinated people in the infection period of Omicron in Tamilnadu state.

Fig. 6 demonstates the stability graph representation of the constructed model in the host population in Tamilnadu with various order of δ. Fig. 6 shows that when the Omicron variant was first discovered, its spread was rapid, and when the government implemented quarantine and vaccination at a high rate, the variant's spread was reduced to a safe level. On March 31, 2022, the state of Tamilnadu discovered that no one had caused the death of Omicron. Covid-19 vaccinations helped people avoid infection with the SARS CoV-2 Omicron variant.

We can observe from Fig. 6 and Table 3 that the infected rate decreases after a rapid spread over a short time with the reproduction number $R_0 < 1$. As a result, the system in the four districts, as well as the entire state of Tamilnadu, is stable.

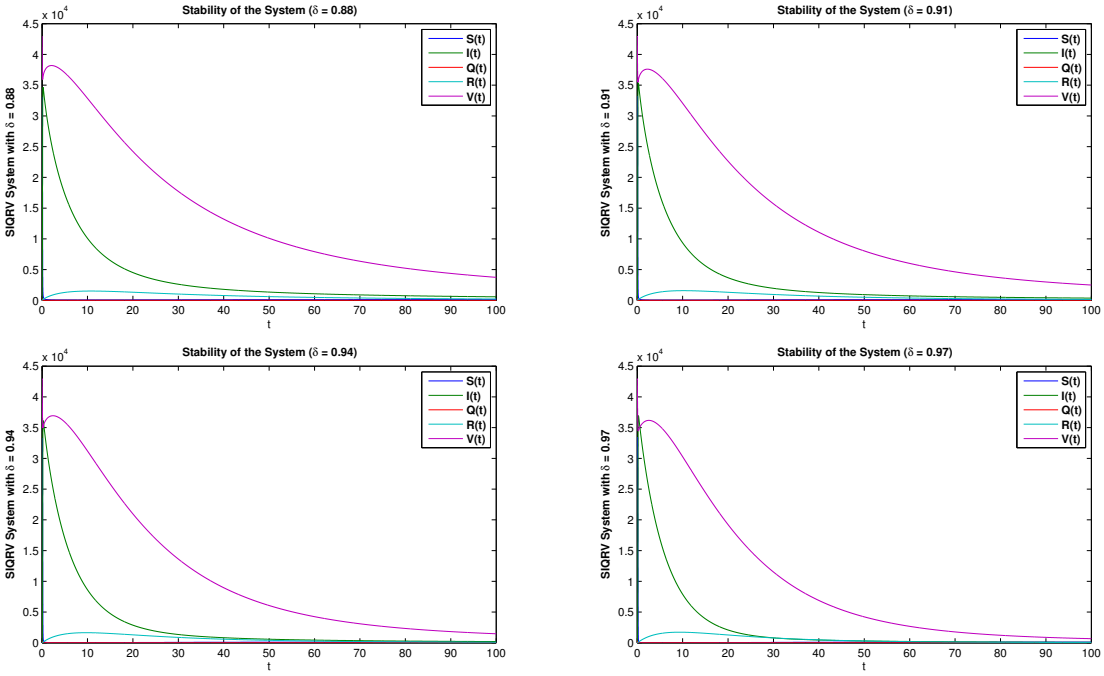

Fig. 6. Stability of the given Omicron System against time t with repect of four different $\delta's$

5 Conclusions

In this study, an SIQRV fractional order mathematical model for COVID-19 was developed. The data acquired from Tamil Nadu together with our mathematical model, suggest that the Omicron variant of COVID-19 infection has been stabilised after few months. This model outperforms other mathematical models by taking into account the nonlinear force of quarantine, vaccine, infection, and care, as well as the right inclusion of valuable parameters. The principles of reproduction number calculated with this model are an outbreak threshold that determined whether or not the disease would go further in Tamilnadu where $R_0 < 1$. These model's fundamentals of positivity and boundedness have been examined and validated. There are infection-free steady-state solutions that are asymptotically stable locally and globally when $R_0 < 1$. Also When $R_0 < 1$ is present, infection-present steady-state solutions that are stable locally are discovered. Finally, the current Omicron variant pandemic data from the Indian state of Tamilnadu is validated.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

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Competing Interests

Authors have declared that no competing interests exist.

References

[1] Chatzarakis GE, et al. A dynamic SI_qIRV Mathematical model with non-linear force of isolation, infection and cure. Nonauton. Dyn. Syst. 2022;9(1):56-67.
Available:https://doi.org/10.1515/msds-2022-0145.

[2] Oke MO, et al. Mathematical modeling and stability analysis of a SIRV epidemic model with non-linear force of infection and treatment. Comm. App. Math. 2019;10(4):717-731.

[3] Rao JPRS, Kumar MN. A dynamic model for infectious diseases: The role of vaccination and treatment. Chaos Solitons and Fractals. 2015;75:34 – 4.
DOI: 10.1016/j.chaos.2015.02.004

[4] Beretta E, Cappasso V. On the general structure of epidemic system: Global stability. Comput. Math. Appl. 1986. 12:677 – 694.
DOI: 10.1016/0898-1221(86)90054-4.

[5] Yang W, et al. Global analysis for a general epidemiological model with vaccination and varying population. J. Math. Anal. Appl. 2010;372:208-223.
DOI: 10.1016/j.jmaa.2010.07.017

[6] Byul Nim Kim, et al. Mathematical model of COVID-19 transmission dynamics in South Korea: The impacts of travel, social distancing and early detection. Processes. 2020;8:1304.
DOI: 10.3390/pr8101304

[7] Daniel Deborah O^*. Mathematical model for the transmission of covid-19 with nonlinear forces of infection and the need for prevention measure in Nigeria. J. Inf. Dis. and Epidemiology. 2020;6(5):1-12.
DOI: 10.23937/2474-3658/1510158

[8] Maski Tomochi, Mitsuo Kono. A mathematical model for COVID-19 pandemic-SIIR model: Effects of asymptomatic individuals. J. Gen. and fam. Med. 2021;22:5-14.
DOI: 10.1002/jgf2.382

[9] Sudhanshu Kumar Biswas, et al. Covid-19 pandemic in India: A mathematical model study. Nonlinear Dyn. Springer Nature B.V; 2020.
DOI: 10.1007/s11071-020-05958-z.

[10] Anwar Zeb, et al. mathematical model for coronavirus disease (COVID-19). Containing Isolation Class. Bio Med Research International. 2019;2020. Article ID 3452402.
Available:https://doi.org/10.1155/2020/3452402.

[11] Ashwin Muniyappan, et al. stability and numerical solutions of second wave mathematical modeling on covid-19 and omicron outbreak strategy of pandemic: Analytical and error analysis of approximate series solutions by using HPM. Mathematics. 2022;343:10.
Available:https://doi.org/10.3390/math10030343.

[12] Bin-Guo Wang, et al. A mathematical model reveals the in the influence of NPIs and vaccination on SARS-CoV-2 Omicron Variant. Research Square.
Available:https://doi.org/10.21203/rs.3.rs-1324280/v1.

[13] Esteva-Peralta L, Velasco-Hernandez JX. M-Matrices and local stability in epidemic model. Math. Comp. Model. 2002;36:491 – 501.
DOI: 10.1016/S0895-7177(02)00178-4

[14] Dickson S, et al. A study on the transmission dynamics of the omicron variant of COVID-19 using nonlinear mathematical models. Comput. Model. Eng. Sci. 2024;139(3):2265-2287.

[15] Fatma Ozkose, et al. Fractional order modelling of Omicron SARS-CoV-2 variant containing heart attack effect using real data from the United Kingdom. Chaos, Solitons and Fractals. 2022;157:111954. Available:https://doi.org/10.1016/j.chaos.2022.111954.

[16] Jianquan Li, et al. Global stability of vaccine-age/staged-structured epidemic models with nonlinear incidence. Electron. J. Qual. Theory Differ. Equ. 2016(18);1–17. DOI: 10.14232/ejqtde.2016.1.18 h

[17] Kumar P, Suat Erturk. A case study of COVID-19 epidemic in India via new generalised Caputo type fractional derivatives. Math. Methods. Appl, Sci. 2021;1-14. DOI: 10.1002/mma.7284

[18] Kurkina ES, Kotsova EM. Mathematical modeling of the propagation of Covid-19 pandemic waves in the world. Comp. Math. And Modeling. 2021;32(2):147-170. DOI: 10.23937/2474-3658/1510158

[19] Li GH, Zhang YX. Dynamic behavior of a modified SIR model in epidemic diseases using non linear incidence rate and treatment. Plos One. 2017;12(4):e0175789. DOI: 10.1371/journal.pone.0175789

[20] Ma Z, Li J. Dynamical modeling and analysis of epidemics. World Scientific Publishing Co.Pte. Ltd. 2009;512. DOI: 10.1142/6799

[21] Pakwan Riyapan, et al. A mathematical model of COVID-19 pandemic: A case study of Bangkok, Thailand. Computational and Mathematical models in Medicine. 2021;6664483:1-11.

[22] Buonomo B, Rionero S. On the stability for SIRS epidemic models with general nonlinear incidence rate. Appl. Mat. Comput. 2010;217:4010-4016. DOI: 10.1016/j.amc.2010.10.007

[23] Derrick W, Grossman S. I Elementary Differential Equations with Applications: Short Course. Addison-Wesley Publishing Company, Philippines; 1976.

[24] Dickson S, et al. SQIRV model for omicron variant with time delay. Aust. J. Math. Anal. Appl., 2022;19(2,Art. 16):16.

[25] Haukkanen P, Tossavainen T. A generalization of descartes rule of signs and fundamental theorem of algebra. Appl. Math. Comput. 2011;218:1203-1207. DOI: 10.1016/j.amc.2011.05.107

[26] La-Salle JP, Lefschetz S. Stability by Liapunovs Direct Method. Academic press, New York; 1961.

[27] Odibat ZM, et al. Generalized Taylors formula. Appl. Math. Comput. 2007;186(1):286–93.

[28] Dickson S, et al. Fractional order mathematical model for B.1.1.529 SARS-Cov-2 Omicron variant with quarantine and vaccination. Int. J. Dyn. Control. 2023;11(5):2215-2231.

[29] Dickson S, et al. A Caputo-type fractional-order SQIRV mathematical model for Omicron variant. Contemp. Math. 2023;4(4):620-636.

[30] Dickson S, et al. Delayed fractional order SEIQI_cRVW mathematical model for Omicron variant. Int. J. Dyn. Control; 2023. Available:https://doi.org/10.1007/s40435-023-01287-2.

[31] Kumar P, et al. A study on the maize streak virus epidemic model by using optimized linearization-based predictor-corrector method in Caputo sense. Chaos, Solitons and Fractals. 2022;158:112067. Available:httpss://doi.org/10.1016/j.chaos.2022.112067.

[32] Lingju Kong, Min Wang. Existence of positive solutions for a fractional compartment system. Electron. J. Qual. Theory Differ. Equ. 2021(60):1–9. Available:https://doi.org/10.14232/ejqtde.2021.1.60

[33] Ndairou F, Torres DFM. Mathematical Analysis of a fractional COVID-19 Model Applied to Wuhan Spain and Portugal. Axioms. 2021;10:135.

[34] Pushpendra Kumar, et al. Some novel mathematical results on the existence and uniqueness of generalized Caputo-type initial value problems with delay. AIMS Mathematics. 2021;7(6):10483–10494.
DOI: 10.3934/math.2022584

[35] Pushpendra Kumar, et al. A study on the maize streak virus epidemic model by using optimized-linearization based predictor-corrector method in Caputo sense. Chaos Solitons and Fractals. 2022;158:112067.
DOI: 10.1016/j.chaos.2022.112067

[36] Shabir Ahmad, et al. Fractional order mathematical modeling of COVID-19 transmission. Chaos, Solitons and Fractals. 2020;139:110256.

[37] Shahram Rezapour, et al. SEIR epidemic model for Covid-19 transmission by Caputo derivative of fractional order. Adv. Differ. Equ. 2020;490.
Available:https://doi.org/10.1186/epjp/s13662-020-02952-y.

[38] Zeeshan Ali, et al. A fractional-order mathematical model for COVID-19 outbreak with the effect of symptomatic and asymptomatic transmissions. Eur. Phys. J. Plus. 2022;137:395.
Available:https://doi.org/10.1140/epjp/s13360-022-02603-z

[39] Podlubny I. Fractional differential equations, mathematics in science and engineering. New York: Academic Press; 1999.

[40] Granas A, Dugundji J. Fixed point theory. Springer. New York; 2015.

[41] Diekmann O, et al. On the definition and computation of the basic reproduction number R_0 in models for infectious disease. J. Math. Biol. 1990;28:365-382.
DOI: 10.1007/BF00178324.

[42] Van den Driessche P, Watmough J. Reproduction number and sub threshold epidemic equilibrium for compartmental models for disease transmission. Math. Biosci. 2002;180:29-48.
DOI: 10.1016/S0025-5564(02)00108-6

[43] Dickson S, et al. Stability analysis of B.1.1.529 SARS-Cov-2 Omicron Variant mathematical model: The impacts of quarantine and vaccination. Nonauton. Dyn. Syst. 2022;9(1):290-306.

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