



Two-step Tensor Splitting Iteration Method for Multi-linear Systems

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2021/v36i930401

Editor(s):

(1) Dr. Dariusz Jacek Jakóbczak, Koszalin University of Technology, Poland.

Reviewers:

(1) Pradip Debnath, Assam University, India.

(2) Şerife büyükköse, Gazi University, Turkey.

(3) Georgy Omelyanov, University of Sonora, Mexico.

Complete Peer review History: <https://www.sdiarticle4.com/review-history/76297>

Received: 04 September 2021

Accepted: 09 November 2021

Published 17 November 2021

Original Research Article

Abstract

In this paper, we construct two-step tensor splitting iteration method for multi-linear systems. Moreover, we present convergence analysis of this method. Finally, we give two numerical examples to show that this new method is more efficient than the existing methods.

Keywords: Two-step; tensor splitting; multi-linear systems.

2010 Mathematics Subject Classification: 15A30; 15A69.

1 Introduction

A high order tensor is a multi-way array whose entries are addressed via multiple indices in the following form:

$$\mathcal{A} = (a_{i_1 i_2 \dots i_m}), a_{i_1 i_2 \dots i_m} \in \mathbb{R}, i_j = 1, 2, \dots, n_j, j = 1, 2, \dots, m,$$

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where \mathbb{R} is the set of real number. If $n_1 = n_2 = \dots = n_m$, then \mathcal{A} is called a square tensor, otherwise it is called a rectangular tensor.

Tensors are higher-order extensions of matrices, and they have wide applications in signal and image processing, continuum physics, higher-order statistics, blind source separation, and especially in exploratory multi-way data analysis ([1]). Hence, tensor analysis and computing have received much attention of researchers in recent decade.

In this paper, we will discuss the following multi-linear system

$$\mathcal{A}x^{m-1} = b, \tag{1.1}$$

where $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{R}^{[m,n]}$ is an order m dimension n tensor, $\mathbb{R}^{[m,n]}$ is the set of order m dimension n tensor, $b \in \mathbb{R}^n$ is a dimension n vector, \mathbb{R}^n is the set of real dimension n vector.

We know an essential problem in pure and applied mathematics is solving various classes of equations. The rapid computation methods of multi-linear systems [2-4] are becoming more and more significant in the field of science and engineering due to their wide applications(see [5-7]). Many research works have been investigated in some literatures on fast solvers for the multi-linear systems (1.1). Ding and Wei [8] proposed some classical iterative methods. Tensor splitting method and its convergence results have been studied by Liu and Li et al. [9]. Some comparison results for splitting iteration for solving multi-linear systems were investigated widely in [10]. Motivated by [9,10], we propose a two-step tensor splitting iteration scheme for solving multi-linear systems.

The remainder of this paper is organized as follows. In Section 2, some basic and useful notations are described simply. In Section 3, a two-step tensor splitting iteration scheme for solving multi-linear systems is proposed. In Section 4, the convergence analysis of the two-step tensor splitting iteration scheme is presented. In Section 5, two numerical examples are given to show the superiority of the new iteration method.

2 Preliminaries

For an m -th order n -dimensional tensor and a vector $x \in \mathbb{R}^n$, $\mathcal{A}x^{m-1}$ is a vector in \mathbb{R}^n with entries

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2, i_3, \dots, i_m=1}^{n, n, \dots, n} a_{i i_2 i_3 \dots i_m} x_{i_2} x_{i_3} \dots x_{i_m}, \quad i = 1, 2, \dots, n.$$

For $A \in \mathbb{R}^{[2,n]}$ and $B \in \mathbb{R}^{[k,n]}$, the matrix-tensor product $C = AB$ is defined by

$$c_{j i_2 \dots i_k} = \sum_{j_2=1}^n a_{j j_2} b_{j_2 i_2 \dots i_k}.$$

For a real m -th order n -dimensional tensor \mathcal{A} and a scalar $\lambda \in \mathbb{C}$, if there exists non-zero vector $x \in \mathbb{C}^n$ such that

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]},$$

where $x^{[m-1]} \in \mathbb{C}^n$ with $(x^{[m-1]})_i = x_i^{m-1}$, $i = 1, 2, \dots, n$, then λ is said to be an eigenvalue of tensor \mathcal{A} and x an eigenvector associated with eigenvalue λ . In particular, if x is real, then λ is also real, and we say (λ, x) is an H -eigenpair of tensor \mathcal{A} . The largest modulus of eigenvalue of tensor \mathcal{A} is called the spectral radius of tensor \mathcal{A} and we denote it by $\rho(\mathcal{A})$.

Definition 2.1. [11] Let $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{R}^{[m,n]}$. \mathcal{A} is said to be a Z -tensor if its off-diagonal entries are all non-positive. \mathcal{A} is said to be an \mathcal{M} -tensor if there exists a nonnegative tensor \mathcal{B} and

a positive real number $c \geq \rho(\mathcal{B})$ such that $\mathcal{A} = c\mathcal{I}_m - \mathcal{B}$. If $c > \rho(\mathcal{B})$, then \mathcal{A} is said to be a strong \mathcal{M} -tensor, where \mathcal{I}_m is identity tensor with all diagonal elements be 1.

Definition 2.2. [12] Let $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{R}^{[m, n]}$, then the majorization matrix $M(\mathcal{A})$ of \mathcal{A} is the $n \times n$ matrix with the entries

$$M(\mathcal{A})_{ij} = a_{ij \dots j}, i, j = 1, 2, \dots, n.$$

Definition 2.3. [10] Let $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{R}^{[m, n]}$, if $M(\mathcal{A})$ is nonsingular and $\mathcal{A} = M(\mathcal{A})\mathcal{I}_m$, then $M(\mathcal{A})^{-1}$ is called the order-2 left-inverse of tensor \mathcal{A} and \mathcal{A} is called left-nonsingular, where \mathcal{I}_m is identity tensor with all diagonal elements be 1.

Definition 2.4. [10] Let $\mathcal{A}, \mathcal{E}, \mathcal{F} \in \mathbb{R}^{[m, n]}$, $\mathcal{A} = \mathcal{E} - \mathcal{F}$ is called a splitting of tensor \mathcal{A} if \mathcal{E} is left-nonsingular, a regular splitting of tensor \mathcal{A} if \mathcal{E} is left-nonsingular with $M(\mathcal{E})^{-1} \geq 0$ and $\mathcal{F} \geq 0$, a weak regular splitting of tensor \mathcal{A} if \mathcal{E} is left-nonsingular with $M(\mathcal{E})^{-1}\mathcal{F} \geq 0$, a convergent splitting of tensor \mathcal{A} if $\rho(M(\mathcal{E})^{-1}\mathcal{F}) < 1$.

3 Two-step Tensor Splitting Iteration Method

Consider two tensor splittings $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1 = \mathcal{E}_2 - \mathcal{F}_2$. Firstly, we describe briefly tensor splitting iterative method for solving multi-linear systems

$$\mathcal{A}x^{m-1} = b.$$

Algorithm 3.1. *Tensor splitting iteration method*

Step 1 Input a tensor \mathcal{A} with splitting $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1$ and a vector b . Given a precision $\varepsilon > 0$ and initial vector x_0 . Set $k := 1$;

Step 2 If $\|\mathcal{A}x_k^{m-1} - b\|_2 < \varepsilon$, then stop; otherwise, go to Step 3;

Step 3

$$x_{k+1} = (M(\mathcal{E}_1)^{-1}\mathcal{F}_1x_k^{m-1} + M(\mathcal{E}_1)^{-1}b)^{[\frac{1}{m-1}]};$$

Step 4 Set $k := k + 1$, return to Step 2.

Where

$$\|\mathcal{A}x_k^{m-1} - b\|_2 = \sqrt{(\mathcal{A}x_k^{m-1} - b)^T(\mathcal{A}x_k^{m-1} - b)}.$$

Let $\mathcal{A} = \mathcal{D} - \mathcal{L} - \mathcal{U}$, where $\mathcal{D} = D\mathcal{I}_m$ and $\mathcal{L} = L\mathcal{I}_m$, D and $-L$ are diagonal and strictly lower triangular parts of $M(\mathcal{A})$, respectively.

When $\mathcal{E}_1 = \frac{1}{\omega}(\mathcal{D} - r\mathcal{L})$, $\mathcal{F}_1 = \frac{1}{\omega}[(1 - \omega)\mathcal{D} + (\omega - r)\mathcal{L}] + \omega\mathcal{U}$, we can get AOR method. Furthermore if $\omega = r$, then we can get SOR method.

Based on Algorithm 3.1, we present two-step tensor splitting iteration method.

Algorithm 3.2. *Two-step tensor splitting iteration method*

Step 1 Input a tensor \mathcal{A} with two splittings $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1 = \mathcal{E}_2 - \mathcal{F}_2$ and a vector b . Given a precision $\varepsilon > 0$ and initial vector x_0 . Set $k := 1$;

Step 2 If $\|\mathcal{A}x_k^{m-1} - b\|_2 < \varepsilon$, then stop; otherwise, go to Step 3;

Step 3

$$x_{k+\frac{1}{2}} = (M(\mathcal{E}_1)^{-1}\mathcal{F}_1x_k^{m-1} + M(\mathcal{E}_1)^{-1}b)^{[\frac{1}{m-1}]},$$

$$x_{k+1} = (M(\mathcal{E}_2)^{-1}\mathcal{F}_2x_{k+\frac{1}{2}}^{m-1} + M(\mathcal{E}_2)^{-1}b)^{[\frac{1}{m-1}]};$$

Step 4 Set $k := k + 1$, return to Step 2.

Let $\mathcal{C} = M(\mathcal{E}_2)^{-1}\mathcal{F}_2$, then

$$\mathcal{C}x_{k+\frac{1}{2}}^{m-1} = M(\mathcal{C})\mathcal{I}_m x_{k+\frac{1}{2}}^{m-1} = M(\mathcal{C})x_{k+\frac{1}{2}}^{[m-1]}.$$

From Step 3 of Algorithm 3.2, we know that

$$x_{k+\frac{1}{2}}^{[m-1]} = M(\mathcal{E}_1)^{-1}\mathcal{F}_1 x_k^{m-1} + M(\mathcal{E}_1)^{-1}b,$$

so

$$\begin{aligned} \mathcal{C}x_{k+\frac{1}{2}}^{m-1} &= M(\mathcal{C})x_{k+\frac{1}{2}}^{[m-1]} = M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1 x_k^{m-1} + M(\mathcal{C})M(\mathcal{E}_1)^{-1}b, \\ x_{k+1} &= (M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1 x_k^{m-1} + M(\mathcal{C})M(\mathcal{E}_1)^{-1}b + M(\mathcal{E}_2)^{-1}b)^{[\frac{1}{m-1}]}. \end{aligned}$$

When

$$\begin{aligned} \mathcal{E}_1 &= \frac{1}{\omega}(\mathcal{D} - r\mathcal{L}), \mathcal{F}_1 = \frac{1}{\omega}[(1 - \omega)\mathcal{D} + (\omega - r)\mathcal{L} + \omega\mathcal{U}], \\ \mathcal{E}_2 &= \frac{1}{\omega}(\mathcal{D} - r\mathcal{U}), \mathcal{F}_2 = \frac{1}{\omega}[(1 - \omega)\mathcal{D} + (\omega - r)\mathcal{U} + \omega\mathcal{L}], \end{aligned}$$

we can get two-step AOR (TAOR) method.

4 Convergence Analysis of Two-step Tensor Splitting Iteration Method

Next we will present the proof of convergence of Algorithm 3.2.

Theorem 4.1. Let $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{R}^{[m, n]}$ and $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1 = \mathcal{E}_2 - \mathcal{F}_2$ be a weak regular splitting and a regular splitting, respectively. If $\mathcal{F}_1 \geq \mathcal{F}_2, \mathcal{F}_2 \neq 0, \mathcal{F}_1 - M(\mathcal{F}_1)\mathcal{I}_m \geq \mathcal{F}_2 - M(\mathcal{F}_2)\mathcal{I}_m, \rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1) < 1$, then there exists a positive Perron vector $x \in \mathbb{R}^n$ such that

$$M(\mathcal{E}_2)^{-1}\mathcal{F}_2 x^{m-1} \leq \rho x^{[m-1]},$$

where $\rho = \rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}\mathcal{S})$, n is a positive integer and $\mathcal{S} \in \mathbb{R}^{[m, n]}$ is a positive tensor.

Proof. Since $\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1) < 1$, we know that there exists a positive integer N such that $\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1) \leq \rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}\mathcal{S}) < 1$ for $n > N$. From $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1$ is a weak regular splitting, we have $M(\mathcal{E}_1)^{-1}\mathcal{F}_1 \geq 0$. While $\mathcal{S} \in \mathbb{R}^{[m, n]}$ is a positive tensor, so $M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}\mathcal{S}$ is positive and irreducible. By the strong Perron-Frobenius theorem [13], there exists a positive Perron vector $x \in \mathbb{R}^n$, such that $(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}\mathcal{S})x^{m-1} = \rho x^{[m-1]}$ for $n > N$, where $\rho = \rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}\mathcal{S})$. Notice that $\rho x^{[m-1]} = \rho\mathcal{I}_m x^{m-1}$, we get

$$\rho\mathcal{I}_m x^{m-1} - \frac{1}{n}\mathcal{S}x^{m-1} = M(\mathcal{E}_1)^{-1}\mathcal{F}_1 x^{m-1},$$

so

$$M(\mathcal{E}_1)(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} = \mathcal{F}_1 x^{m-1},$$

From $\mathcal{A} = \mathcal{E}_1 - \mathcal{F}_1$, we know that $M(\mathcal{A}) = M(\mathcal{E}_1) - M(\mathcal{F}_1)$, so

$$\begin{aligned} M(\mathcal{A})(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} &= (M(\mathcal{E}_1) - M(\mathcal{F}_1))(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} \\ &= (M(\mathcal{E}_1)(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} - (M(\mathcal{F}_1)(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} \\ &= \mathcal{F}_1 x^{m-1} - (M(\mathcal{F}_1)(\rho\mathcal{I}_m - \frac{1}{n}\mathcal{S})x^{m-1} \\ &= [\mathcal{F}_1 - M(\mathcal{F}_1)\mathcal{I}_m]x^{m-1} + M(\mathcal{F}_1)\mathcal{I}_m x^{m-1} - \rho M(\mathcal{F}_1)\mathcal{I}_m x^{m-1} + \frac{1}{n}M(\mathcal{F}_1)\mathcal{S}x^{m-1} \\ &= [\mathcal{F}_1 - M(\mathcal{F}_1)\mathcal{I}_m]x^{m-1} + (1 - \rho)M(\mathcal{F}_1)\mathcal{I}_m x^{m-1} + \frac{1}{n}M(\mathcal{F}_1)\mathcal{S}x^{m-1} \end{aligned}$$

From $\mathcal{A} = \mathcal{E}_2 - \mathcal{F}_2$, we know that $M(\mathcal{A}) = M(\mathcal{E}_2) - M(\mathcal{F}_2)$, so

$$M(\mathcal{A})(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} = [M(\mathcal{E}_2) - M(\mathcal{F}_2)](\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1}.$$

From $\mathcal{F}_1 \geq \mathcal{F}_2$ and Definition 2.2, we know that $M(\mathcal{F}_1) \geq M(\mathcal{F}_2)$. From $\mathcal{F}_1 - M(\mathcal{F}_1)\mathcal{I}_m \geq \mathcal{F}_2 - M(\mathcal{F}_2)\mathcal{I}_m$, we get

$$\begin{aligned} [M(\mathcal{E}_2) - M(\mathcal{F}_2)](\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} &= [\mathcal{F}_1 - M(\mathcal{F}_1)\mathcal{I}_m]x^{m-1} + (1 - \rho)M(\mathcal{F}_1)\mathcal{I}_m x^{m-1} + \frac{1}{n}M(\mathcal{F}_1)Sx^{m-1} \\ &\geq [\mathcal{F}_2 - M(\mathcal{F}_2)\mathcal{I}_m]x^{m-1} + (1 - \rho)M(\mathcal{F}_2)\mathcal{I}_m x^{m-1} + \frac{1}{n}M(\mathcal{F}_2)Sx^{m-1} \end{aligned}$$

i.e.,

$$M(\mathcal{E}_2)(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} - M(\mathcal{F}_2)(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} \geq \mathcal{F}_2 x^{m-1} - M(\mathcal{F}_2)(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1},$$

so

$$M(\mathcal{E}_2)(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} \geq \mathcal{F}_2 x^{m-1},$$

i.e.,

$$\mathcal{F}_2 x^{m-1} \leq M(\mathcal{E}_2)(\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1}.$$

From $\mathcal{A} = \mathcal{E}_2 - \mathcal{F}_2$ is a regular splitting, we know that $M(\mathcal{E}_2)^{-1} \geq 0$ and $\mathcal{F}_2 \geq 0$, hence

$$M(\mathcal{E}_2)^{-1}\mathcal{F}_2 x^{m-1} \leq (\rho\mathcal{I}_m - \frac{1}{n}S)x^{m-1} \leq \rho\mathcal{I}_m x^{m-1} = \rho x^{[m-1]}.$$

□

Theorem 4.2. *If all the conditions of Theorem 3.1 hold and $\mathcal{C} = M(\mathcal{E}_2)^{-1}\mathcal{F}_2$ is left-nonsingular, then*

$$\rho(M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1) \leq [\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1)]^2$$

and

$$\rho(M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1) < 1.$$

Proof. From the proof of Theorem 4.1, we know that there exists a positive integer N and a positive Perron vector $x \in \mathbb{R}^n$, such that

$$(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}S)x^{m-1} = \rho x^{[m-1]}$$

for $n > N$, where $\rho = \rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}S)$. So

$$M(\mathcal{C})(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}S)x^{m-1} = \rho M(\mathcal{C})x^{[m-1]} = \rho M(\mathcal{C})\mathcal{I}_m x^{m-1}.$$

Since $\mathcal{C} = M(\mathcal{E}_2)^{-1}\mathcal{F}_2$ is left-nonsingular, then $M(\mathcal{C})\mathcal{I}_m = \mathcal{C}$, we get

$$M(\mathcal{C})(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}S)x^{m-1} = \rho M(\mathcal{C})\mathcal{I}_m x^{m-1} = \rho \mathcal{C} x^{m-1} = \rho M(\mathcal{E}_2)^{-1}\mathcal{F}_2 x^{m-1}.$$

From Theorem 4.1, we know that

$$M(\mathcal{E}_2)^{-1}\mathcal{F}_2 x^{m-1} \leq \rho x^{[m-1]},$$

so

$$M(\mathcal{C})(M(\mathcal{E}_1)^{-1}\mathcal{F}_1 + \frac{1}{n}S)x^{m-1} = \rho M(\mathcal{E}_2)^{-1}\mathcal{F}_2 x^{m-1} \leq \rho^2 x^{[m-1]}.$$

When $n \rightarrow \infty$, we have

$$M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1x^{m-1} \leq [\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1)]^2x^{[m-1]},$$

so

$$\rho(M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1) \leq [\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1)]^2.$$

From $\rho(M(\mathcal{E}_1)^{-1}\mathcal{F}_1) < 1$, we get $\rho(M(\mathcal{C})M(\mathcal{E}_1)^{-1}\mathcal{F}_1) < 1$.

□

5 Examples

In this section, two numerical examples are given to show the effectiveness of two-step tensor splitting iteration method.

All the numerical experiments have been carried out by MATLAB R2011b 7.1.3. Iterations are terminated when the norm of the residual vector (denoted by 'RES') $\text{RES} = \|\mathcal{A}x_k^{m-1} - b\|_2 < 10^{-11}$.

Example 5.1. Consider the multi-linear systems with a strong \mathcal{M} -tensor

$$\mathcal{A} = 864.4895\mathcal{I}_3 - \mathcal{B},$$

where $\mathcal{B} \in \mathbb{R}^{[3,5]}$ is a nonnegative tensor with $b_{i_1 i_2 i_3} = |\tan(i_1 + i_2 + i_3)|$.

Table 1. Numerical results for Example 5.1 when $r = 2.3, \omega = 0.99$

method	SOR	AOR	TAOR
IT	380	307	133
CPU	1.5005	1.0568	0.7945

Example 5.2. Consider the multi-linear systems with a strong \mathcal{M} -tensor

$$\mathcal{A} = 9\mathcal{I}_3 - \mathcal{B},$$

where $\mathcal{B} \in \mathbb{R}^{[3,3]}$ is a nonnegative tensor with $b_{i_1 i_2 i_3} = |\sin(i_1 + i_2 + i_3)|$.

Table 2. Numerical results for Example 5.2 when $r = 2.3, \omega = 0.99$

method	SOR	AOR	TAOR
IT	51	39	23
CPU	0.2984	0.2325	0.2182

In Tables 1 and 2, the number of iteration steps (denoted by IT) and the elapsed CPU time in seconds (denoted by CPU) are listed for SOR, AOR and TAOR methods when $r = 2.3, \omega = 0.99$, respectively. From the numerical results, we can see that TAOR method requires less iteration steps and CPU time than SOR and AOR methods, so TAOR method is more efficient than SOR and AOR methods.

6 Conclusion

In this paper, we construct two-step tensor splitting iteration method for multi-linear systems and present convergence analysis of this method. Finally, we give two numerical examples to show that this new method is more efficient than the existing ones.

Acknowledgement

This work was supported by the National Nature Science Foundation of China (Grants No. 12171307), the Science and Technology Program of Shandong Universities (No. J16LI04) and the Advanced Talents Foundation of Qingdao Agricultural University (No. 1120068).

Competing Interests

Authors have declared that no competing interests exist.

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